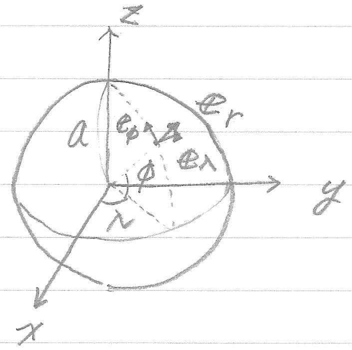
Rossby Waves

回転球面上の2次元非発散運動 — Rossby波



単位ベクトル

単位ベクトルの微分

微分演算子

1. 基礎方程式

$$\begin{cases} u = a\cos\phi \dfrac{D\lambda}{Dt} \\ v = a\dfrac{D\phi}{Dt} \end{cases}$$

速度ベクトル：

$$\boldsymbol{u} = u\,\boldsymbol{e}_\lambda + v\,\boldsymbol{e}_\phi \qquad (1\text{-}1)$$

◎ 運動方程式：

D_λ
D_ϕ

$\boldsymbol{e}_\lambda$:

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} - 2\Omega\sin\phi\, v = -\frac{1}{\rho}\frac{1}{a\cos\phi}\frac{\partial p}{\partial \lambda} + F_\lambda \qquad (1\text{-}2)$$

$\boldsymbol{e}_\phi$:

$$\frac{Dv}{Dt} + \frac{u^2\tan\phi}{a} + 2\Omega\sin\phi\, u = -\frac{1}{\rho}\frac{1}{a}\frac{\partial p}{\partial \phi} + F_\phi \qquad (1\text{-}3)$$

ただし、

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda} + v\frac{1}{a}\frac{\partial}{\partial \phi}$$

◎ 連続の式 (ρ = 一定　$\nabla\cdot\boldsymbol{u} = 0$)

$$\frac{1}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\cos\phi\, v) = 0 \qquad (1\text{-}4)$$

NO.

from cont'

$$M = a\cos\phi\,(u + a\cos\phi\,\Omega)$$

$$\frac{DM}{Dt} = \left(\frac{\partial M}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial M}{\partial \lambda} + \frac{v}{a}\frac{\partial M}{\partial \phi}\right) + M\left\{\frac{1}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(\cos\phi\, v)\right\}$$

$$= \frac{\partial M}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial Mu}{\partial \lambda} + \frac{v\cos\phi}{a\cos\phi}\frac{\partial M}{\partial \phi} + \frac{M}{a\cos\phi}\frac{\partial}{\partial \phi}(v\cos\phi)$$

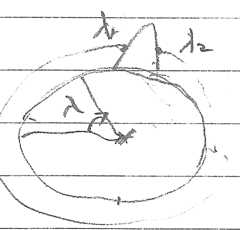
$$= \frac{\partial M}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial Mu}{\partial \lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(Mv\cos\phi)$$

zonal mean

$$\frac{\partial \overline{M}}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left[(\overline{M}\,\overline{v} + \overline{M'v'})\cos\phi\right] = -\frac{1}{\rho}\frac{\partial p}{\partial \lambda} + a\cos\phi\,\overline{F_\lambda}$$

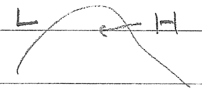
$$F_\lambda = -ku$$

① 山



$$-\frac{1}{\rho_0}\int_0^{2\pi}\frac{\partial p}{\partial \lambda}\,d\lambda = -\int_0^{\lambda_1} dp - \int_{\lambda_2}^{0} dp = -p(\lambda_1) + p(0) - p(0) + p(\lambda_2)$$

$$= p(\lambda_2) - p(\lambda_1)$$



大気が山を西向きに押す
山が大気を東向きに押す

② 地表 マサツ

西風 $u > 0$ 大気が地球を東向きにまわす
地球が大気を西向きにまわす

東風

○ 運動方程式 e_λ 成分は 角運動量保存 (Note-1)

$$\frac{\partial}{\partial t}\{\underbrace{a\cos\phi\,(u + a\cos\phi\,\Omega)}_{M}\} = -\frac{1}{\rho}\frac{\partial p}{\partial \lambda} + a\cos\phi\, F_\lambda \tag{1-5}$$

フラックス形式

$$\frac{\partial M}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial (Mu)}{\partial \lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(Mv\cos\phi) = -\frac{1}{\rho}\frac{\partial p}{\partial \lambda} + a\cos\phi\, F_\lambda \tag{1-6}$$

○ 流線関数 ψ ($\leftarrow$ (1-4))

$\nabla \rightarrow$ Note2

$$(u, v, 0)^T = \boldsymbol{u} = \boldsymbol{e}_r \times \nabla\psi$$

$$\begin{cases} u = -\dfrac{1}{a}\dfrac{\partial \psi}{\partial \phi} = -\dfrac{\sqrt{1-\mu^2}}{a}\dfrac{\partial \psi}{\partial \mu} \\[2ex] v = \dfrac{1}{a\cos\phi}\dfrac{\partial \psi}{\partial \lambda} = \dfrac{1}{a\sqrt{1-\mu^2}}\dfrac{\partial \psi}{\partial \lambda} \end{cases} \tag{1-7}$$

$(w = 0)$ $\qquad \mu = \sin\phi$

○ vector invariant form (Note-2)

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\nabla\times\boldsymbol{u} + f\boldsymbol{e}_r)\times\boldsymbol{u} + \nabla\frac{|\boldsymbol{u}|^2}{2} = -\frac{1}{\rho}\nabla p + \boldsymbol{F} \tag{1-8}$$

○ 渦度 ζ

$$\zeta = \boldsymbol{e}_r\cdot(\nabla\times\boldsymbol{u}) = \frac{1}{a\cos\phi}\left[\frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \phi}(\cos\phi\, u)\right]$$

$$= \frac{1}{a^2\cos^2\phi}\frac{\partial^2\psi}{\partial \lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi\frac{\partial \psi}{\partial \phi}\right)$$

$$\zeta = \Delta_2 \psi \tag{1-9}$$

$$\Delta_2 = \frac{1}{a^2\cos^2\phi}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial}{\partial\phi}\right)$$

$$= \frac{1}{a^2}\left[\frac{1}{1-\mu^2}\frac{\partial^2}{\partial\lambda^2} + \frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial}{\partial\mu}\right\}\right]$$

◎ 渦度方程式 (Note・2)

$\mathbf{e}_r \cdot (\nabla \times (1\text{-}8))$

$$\frac{D}{Dt}(\zeta + f) = \frac{1}{a\cos\phi}\left[\frac{\partial F_\phi}{\partial\lambda} - \frac{\partial}{\partial\phi}(\cos\phi\, F_\lambda)\right] = F \tag{1-10}$$

○ Jacobian. (Note・3)

$$(\mathbf{u}\cdot\nabla)(\zeta+f) = \frac{1}{a^2}J(\psi, \zeta+f)$$

$$= \frac{1}{a^2}\left\{\frac{\partial\psi}{\partial\lambda}\frac{\partial}{\partial\mu}(\zeta+f) - \frac{\partial\psi}{\partial\mu}\frac{\partial}{\partial\lambda}(\zeta+f)\right\}$$

$(f = 2\Omega\mu)$

$(1\text{-}10) \rightarrow$

$$\frac{\partial\zeta}{\partial t} + \frac{1}{a^2}J(\psi, \zeta+f) = F \tag{1-11}$$

$$\left(\begin{array}{l} \zeta = \Delta_2\psi, \quad f = 2\Omega\mu \\ J(\alpha,\beta) = \frac{\partial\alpha}{\partial\lambda}\frac{\partial\beta}{\partial\mu} - \frac{\partial\alpha}{\partial\mu}\frac{\partial\beta}{\partial\lambda} \end{array}\right.$$

2. 波－帯状平均相互作用

◉ 帯状平均とズレ. $(\varepsilon \ll 1)$

$$\overline{\circledcirc} = \frac{1}{2\pi}\int_0^{2\pi} \circledcirc \, d\lambda \qquad (2\text{-}1)$$

$$\left.\begin{aligned}
\psi &= \overline{\psi} + \varepsilon\psi' + \varepsilon^2\psi^{(2)} + \cdots \\
u &= \overline{u} + \varepsilon u' + \varepsilon^2 u^{(2)} + \cdots \\
v &= \varepsilon v' + \varepsilon^2 v^{(2)} + \cdots \\
\zeta &= \overline{\zeta} + \varepsilon\zeta' + \varepsilon^2\zeta^{(2)} + \cdots
\end{aligned}\right\} \qquad (2\text{-}2)$$

$$F = \overline{F} + \varepsilon F' + \varepsilon^2 F^{(2)} + \cdots$$

$$f = f$$

○ 帯状平均量, $\bigcirc'$ 量 $\cos\phi$

$$(1\text{-}7) \rightarrow \quad \overline{u} = -\frac{\sqrt{1-\mu^2}}{a}\frac{\partial\overline{\psi}}{\partial\mu} \qquad (2\text{-}3)$$

$$(1\text{-}7) \rightarrow \quad \overline{v} = 0 \qquad (2\text{-}4)$$

$$(1\text{-}9) \rightarrow \quad \overline{\zeta} = \frac{1}{a^2}\frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial\overline{\psi}}{\partial\mu}\right\} \qquad (2\text{-}5)$$

$$\rightarrow \quad u' = -\frac{\sqrt{1-\mu^2}}{a}\frac{\partial\psi'}{\partial\mu} \qquad (2\text{-}6)$$

$$\rightarrow \quad v' = \frac{1}{a\sqrt{1-\mu^2}}\frac{\partial\psi'}{\partial\lambda} \qquad (2\text{-}7)$$

$$\rightarrow \quad \zeta' = \frac{1}{a^2}\left[\frac{1}{1-\mu^2}\frac{\partial^2}{\partial\lambda^2} + \frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial}{\partial\mu}\right\}\right]\psi' \qquad (2\text{-}8)$$

◎ O(0) 渦度方程式

$$\frac{\partial \bar{\zeta}}{\partial t} = \bar{F} \quad (2\text{-}9)$$

$\bar{F}=0$ で $\bar{\zeta}$ の時間変化なし.

◎ O(ε) 渦度方程式

$$\frac{\partial \zeta'}{\partial t} + \frac{1}{a^2} J(\bar{\psi}, \zeta') + \frac{1}{a^2} J(\psi', \bar{q}) = F' \quad (2\text{-}10)$$

$$\bar{q} = \bar{\zeta} + f$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}\right)\zeta' + v'\frac{1}{a}\frac{\partial \bar{q}}{\partial \phi} = F' \quad (2\text{-}10)\ (N\text{-}3)$$

○ O(ε) エンストロフィー : $\frac{\zeta'^2}{2}$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}\right)\frac{\zeta'^2}{2} + \zeta' v'\frac{1}{a}\frac{\partial \bar{q}}{\partial \phi} = \zeta' F' \quad (2\text{-}11)$$

帯状平均

$$\frac{\partial}{\partial t}\overline{\frac{\zeta'^2}{2}} + \overline{\zeta' v'}\frac{1}{a}\frac{\partial \bar{q}}{\partial \phi} = \overline{\zeta' F'} \quad (2\text{-}12)\ (N\text{-}3)$$

◎ $O(\varepsilon^2)$ 渦度方程式

$$\frac{\partial \zeta^{(2)}}{\partial t} + \frac{1}{a^2} J(\overline{\psi}, \zeta^{(2)}) + \frac{1}{a^2} J(\psi^{(2)}, \overline{q}) + \frac{1}{a^2} J(\psi', \zeta') = F^{(2)} \qquad (2\text{-}13)$$

$$\left(\frac{\partial}{\partial t} + \overline{u} \frac{1}{a\cos\phi} \frac{\partial}{\partial \lambda}\right) \zeta^{(2)} + v^{(2)} \frac{1}{a} \frac{\partial \overline{q}}{\partial \phi} + \left(u' \frac{1}{a\cos\phi} \frac{\partial}{\partial \lambda} + v' \frac{1}{a} \frac{\partial}{\partial \phi}\right) \zeta' = F^{(2)} \qquad (2\text{-}14)$$

ここだけ フラックス形式

$$\left(\frac{\partial}{\partial t} + \overline{u} \frac{1}{a\cos\phi} \frac{\partial}{\partial \lambda}\right) \zeta^{(2)} + v^{(2)} \frac{1}{a} \frac{\partial \overline{q}}{\partial \phi} + \frac{1}{a\cos\phi} \left[\frac{\partial}{\partial \lambda}(\zeta' u') + \frac{\partial}{\partial \phi}(\cos\phi\, \zeta' v')\right] = F^{(2)} \qquad (2\text{-}14)'$$

$\overline{(2\text{-}14)'}$

$$\frac{\partial}{\partial t} \overline{\zeta^{(2)}} + \frac{1}{a\cos\phi} \frac{\partial}{\partial \phi}\left(\cos\phi\, \overline{\zeta' v'}\right) = \overline{F^{(2)}} \qquad (2\text{-}15)$$

(p.2, ℓ-2) → $\overline{\zeta^{(2)}} = -\frac{1}{a\cos\phi} \frac{\partial}{\partial \phi}\left(\cos\phi\, \overline{u^{(2)}}\right)$

(1-10) → $\overline{F^{(2)}} = -\frac{1}{a\cos\phi} \frac{\partial}{\partial \phi}\left(\cos\phi\, \overline{F_\lambda^{(2)}}\right)$

$$\frac{\partial \overline{u^{(2)}}}{\partial t} - \overline{\zeta' v'} = \overline{F_\lambda^{(2)}} \qquad (2\text{-}16)$$

◎ $O(\varepsilon)$ エンストロフィー方程式のフラックス形式

(2-11): $\left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\right)\frac{\zeta'^2}{2} + \zeta' v' \frac{1}{a}\frac{\partial \bar{q}}{\partial\phi} = \zeta' F'$

$$\zeta' v' = \left\{\frac{1}{a^2\cos^2\phi}\frac{\partial^2\psi'}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\psi'}{\partial\phi}\right)\right\}\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}$$

$$= \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right)^2 - \frac{1}{2}\left(\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right)^2\right\}$$
$$+ \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left\{\cos^2\phi\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right\} \qquad (2\text{-}17)$$

(2-11) →

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\right)\frac{\zeta'^2}{2}$$
$$+ \frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right)^2 - \frac{1}{2}\left(\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right)^2\right\}\right.$$
$$\left. + \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left\{\cos^2\phi\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right\}\right]$$
$$= \zeta' F'$$

$$\div\left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\cdot\frac{1}{\cos\phi}\right) \qquad + \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\right)^{-1}\frac{\zeta'^2}{2}\cos\phi\,\bar{u}\right\}$$

$$\frac{\partial}{\partial t}\left\{\left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\right)^{-1}\frac{\zeta'^2}{2}\cos\phi\right\} - \frac{\zeta'^2}{2}\cos\phi\frac{\partial}{\partial t}\left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\right)^{-1}$$

$O(\varepsilon^3)$

$$+ \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right)^2 - \frac{1}{2}\left(\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right)^2\right\}\cos\phi$$

$$+ \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left\{\cos^2\phi\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right\}$$

$$= \left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\right)^{-1}\zeta' F'\cos\phi \qquad (2\text{-}18)$$

$$\nabla \cdot \mathbb{A} = \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}A_\lambda + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\, A_\phi)$$

⑩ 波の活動度に関する保存則

$$\therefore \boxed{\frac{\partial A}{\partial t} + \nabla\cdot\mathbb{F} = \left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\right)^{-1}\zeta' F'\cos\phi} \quad (+\,O(\varepsilon^3)) \qquad (2\text{-}19)$$

$$A = \left(\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}\right)^{-1}\frac{\zeta'^2}{2}\cos\phi \qquad (2\text{-}20)$$

$$\mathbb{F} = \begin{pmatrix} A\bar{u} + \frac{1}{2}\left\{\left(\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right)^2 - \left(\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right)^2\right\}\cos\phi \\ \frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\cos\phi \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} A\bar{u} + \frac{1}{2}(v'^2 - u'^2)\cos\phi \\ u'v'\cos\phi \\ 0 \end{pmatrix} \qquad (2\text{-}21)$$

Plumb (1985) JAS 42, 217-229.
conservation relation for wave activity

A : density of wave activity

$\mathbb{F}$: total flux of wave activity

advective flux : $(A\bar{u}, 0)^T$

radiative flux : $\cos\phi\left(\frac{1}{2}(v'^2-u'^2),\ u'v'\right)^T$

$$\boxed{\nabla\cdot\mathbb{F} = \zeta' v'\cos\phi}$$

(friction の F と混同しないこ

② 帯状平均流 : $\bar{u}^{(2)}$

(2-16) : $\dfrac{\partial \bar{u}^{(2)}}{\partial t} - \overline{\zeta' v'} = \overline{F_\lambda}^{(2)}$　　$\dfrac{1}{a\cos\phi}\dfrac{\partial}{\partial\phi}(\cos^2\phi\,\overline{u'v'})$

$$\frac{\partial \bar{u}^{(2)}}{\partial t} = \frac{1}{\cos\phi}\overline{\nabla\cdot\mathbb{F}} + \overline{F_\lambda}^{(2)}$$

$$\therefore\ \boxed{\frac{\partial}{\partial t}\bar{u}^{(2)}\cos\phi = \overline{\nabla\cdot\mathbb{F}} + \overline{F_\lambda}^{(2)}\cos\phi} \qquad (2\text{-}21)$$

↳ $\dfrac{1}{a\cos\phi}\dfrac{\partial}{\partial\phi}(\cos\phi\ \overline{u'\cos\phi\, v'})$

"擾乱は、角運動量を保存的に輸送する。"

局所的に定常で ($\frac{\partial A}{\partial t}=0$)、保存的な ($F'=0$) 波に対して、波の活動度の全フラックスは非発散 ($\nabla\cdot\mathbb{F}=0$) である。

↓

その時、波による帯状角運動量の変化はない。

"非加速定理"

(2-19)、(2.21) より

$$\boxed{\frac{\partial}{\partial t}(A + \bar{u}^{(2)}\cos\phi) = \left(\frac{1}{a}\frac{\partial}{\partial\phi}\right)^{-1}\overline{\zeta' F'}\cos\phi + \overline{F_\lambda}^{(2)}} \qquad (2\text{-}22)$$

3. WKB 近似

◎ 位相関数 Θ real

$$\psi = \sum \varepsilon^n \psi_n(\Lambda, \Phi, T)\, e^{i\Theta(\Lambda,\Phi,T)/\varepsilon} \qquad (3\text{-}1)$$

slow variables $\Lambda = \varepsilon\lambda$, $\Phi = \varepsilon\phi$, $T = \varepsilon t$

$0 < \varepsilon \ll 1$

◎ 局所分散関係

(3-1) → (2-10) : $O(\varepsilon^0)$ Note-6

$$\left[-i\left(\frac{\partial\Theta}{\partial T} + \frac{\bar{u}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\right)\left\{\frac{1}{a^2\cos^2\Phi}\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \frac{1}{a^2}\left(\frac{\partial\Theta}{\partial\Phi}\right)^2\right\} + \frac{i}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\hat{\beta}\right]\psi_0 e^{i\Theta/\varepsilon} = 0 \qquad (3\text{-}2)$$

$\bar{u}(\Phi)$ および $\hat{\beta}(\Phi) = \frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}$ slow variable only 仮定.

○ 局所波数, 振動数

$$\left.\begin{aligned} k &= \frac{1}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda} \\ l &= \frac{1}{a}\frac{\partial\Theta}{\partial\Phi} \\ \omega &= -\frac{\partial\Theta}{\partial T} \end{aligned}\right\} \qquad (3\text{-}3)$$

(3-3) → (3-2)

$$\omega = \bar{u}k - \frac{k\hat{\beta}}{k^2+l^2} \equiv \omega(k, l, \Phi) \qquad (3\text{-}4)$$

我k,

$$\omega(k\cos\Phi, l, \Phi) = \frac{\bar{u}}{\cos\Phi}(k\cos\Phi) - \frac{(k\cos\Phi)\cdot\frac{\hat{\beta}}{\cos\Phi}}{\frac{(k\cos\Phi)^2}{\cos^2\Phi} + l^2} \quad (3\text{-}5)$$

$$\hat{\omega} = \omega - \bar{u}k = -\frac{k\hat{\beta}}{k^2+l^2} \quad (3\text{-}6)$$

◎ 群速度 Note・7

$$C_{gx} \equiv \frac{\partial \omega}{\partial(k\cos\Phi)}\cdot\cos\Phi$$

$$= \bar{u} + \frac{k^2-l^2}{(k^2+l^2)^2}\hat{\beta} = \bar{u} + \frac{k^2-l^2}{k^2}\cdot\frac{\hat{\omega}^2}{\hat{\beta}} \quad (3\text{-}7)$$

$$C_{g\phi} \equiv \frac{\partial\omega}{\partial l}$$

$$= \frac{2kl}{(k^2+l^2)^2}\hat{\beta} = \frac{2l}{k}\cdot\frac{\hat{\omega}^2}{\hat{\beta}} \quad (3\text{-}8)$$

◎ 波数保存則

$$\left.\begin{aligned} \frac{\partial}{\partial T}(k\cos\Phi) &= \frac{1}{a}\frac{\partial^2\Theta}{\partial T\partial\Lambda} = -\frac{1}{a}\frac{\partial\omega}{\partial\Lambda} \\ \frac{\partial l}{\partial T} &= \frac{1}{a}\frac{\partial^2\Theta}{\partial T\partial\Phi} = -\frac{1}{a}\frac{\partial\omega}{\partial\Phi} \\ \frac{\partial}{\partial\Phi}(k\cos\Phi) &= \frac{1}{a}\frac{\partial^2\Theta}{\partial\Phi\partial\Lambda} = \frac{\partial l}{\partial\Lambda} \end{aligned}\right\} \quad (3\text{-}9)$$

$$\frac{\partial}{\partial T}(k\cos\Phi) = -\frac{1}{a}\frac{\partial\omega}{\partial\Lambda}$$

$$= -\frac{1}{a}\left(\frac{\partial\omega}{\partial(k\cos\Phi)}\frac{\partial(k\cos\Phi)}{\partial\Lambda} + \frac{\partial\omega}{\partial l}\overbrace{\frac{\partial l}{\partial\Lambda}}^{=\frac{\partial}{\partial\Phi}(k\cos\Phi)} + \frac{\partial\omega}{\partial\Lambda}\right)$$

$$= -C_{gx}\frac{1}{a\cos\phi}\frac{\partial(k\cos\Phi)}{\partial\Lambda} - C_{g\phi}\frac{1}{a}\frac{\partial}{\partial\Phi}(k\cos\Phi) - \frac{1}{a}\frac{\partial\omega}{\partial\Lambda} \quad (3\text{-}10)$$

$$\frac{\partial l}{\partial T} = -\frac{1}{a}\frac{\partial \omega}{\partial \Phi}$$

$$= -\frac{1}{a}\left(\frac{\partial \omega}{\partial (k\cos\Phi)}\overset{\overset{\frac{\partial l}{\partial \lambda}}{\parallel}}{\frac{\partial (k\cos\Phi)}{\partial \Phi}} - \frac{\partial \omega}{\partial l}\frac{\partial l}{\partial \Phi} - \frac{\partial \omega}{\partial \Phi}\right)$$

$$= -c_{g\lambda}\frac{1}{a\cos\Phi}\frac{\partial l}{\partial \lambda} - c_{g\phi}\frac{1}{a}\frac{\partial l}{\partial \Phi} - \frac{1}{a}\frac{\partial \omega}{\partial \Phi} \qquad (3\text{-}11)$$

$$\frac{\partial \omega}{\partial T} = \frac{\partial \omega}{\partial (k\cos\Phi)}\overset{-\frac{1}{a}\frac{\partial \omega}{\partial \lambda}}{\frac{\partial (k\cos\Phi)}{\partial T}} + \frac{\partial \omega}{\partial l}\overset{-\frac{1}{a}\frac{\partial \omega}{\partial \Phi}}{\frac{\partial l}{\partial T}} + \frac{\partial \omega}{\partial T}$$

$$= -c_{g\lambda}\frac{1}{a\cos\Phi}\frac{\partial \omega}{\partial \lambda} - c_{g\phi}\frac{1}{a}\frac{\partial \omega}{\partial \Phi} + \frac{\partial \omega}{\partial T} \qquad (3\text{-}12)$$

$\omega(\Phi)^{\text{only}}$

$$\therefore\quad \left.\begin{aligned} D_T(k\cos\Phi) &= -\frac{1}{a}\frac{\partial \omega}{\partial \lambda} = 0 \\ D_T\, l &= -\frac{1}{a}\frac{\partial \omega}{\partial \Phi}^{\neq 0} \\ D_T\, \omega &= \frac{\partial \omega}{\partial T} = 0 \end{aligned}\right\} \qquad (3\text{-}13)$$

ただし、

$$D_T = \frac{\partial}{\partial T} + c_{g\lambda}\frac{1}{a\cos\Phi}\frac{\partial}{\partial \lambda} + c_{g\phi}\frac{1}{a}\frac{\partial}{\partial \Phi}$$

$\mathbf{c}_g = (c_{g\lambda}, c_{g\phi})^T$ に沿って $k\cos\Phi$ と ω は不変.

② 転移緯度 (turning latitude)

"波"存在緯度は ω, k に対して $l^2>0$ である条件として与えられる。($k>0$ とする)

(3-6) →

$$l^2 = -\frac{k\hat{\beta}}{\omega - \bar{u}k} - k^2 > 0$$

$$\therefore \quad -\frac{\hat{\beta}}{\omega - \bar{u}k} > k \qquad (3\text{-}14)$$

(3-13)

波線（波束の伝播経路）に沿って $k\cos\Phi = \mathrm{const.} = k_M$

であることより、

$$-\frac{2\Omega_M \cos^2\Phi}{\omega - \frac{\bar{u}}{\cos\Phi}k_M} > k_M > 0 \qquad (3\text{-}15)$$

ただし、$2\Omega_M \equiv \dfrac{\hat{\beta}}{\cos\Phi} = \dfrac{1}{\cos\Phi}\dfrac{1}{a}\dfrac{\partial \bar{q}}{\partial \Phi}$

$$= \frac{1}{\cos\Phi}\frac{1}{a}\frac{\partial}{\partial\Phi}(\bar{\zeta} + 2\Omega\sin\Phi)$$

$$= \frac{1}{a\cos\Phi}\frac{\partial}{\partial\Phi}\bar{\zeta} + \frac{2\Omega}{a}$$

$$= -\frac{1}{a\cos\Phi}\frac{\partial}{\partial\Phi}\left[\frac{1}{a\cos\Phi}\frac{\partial}{\partial\Phi}(\cos\Phi\,\bar{u})\right] + \frac{2\Omega}{a}$$

$\Phi \to \pm\frac{\pi}{2}$ において $\Omega_M < \infty$、$\dfrac{\bar{u}}{\cos\Phi} < \infty$ なので

不等式 左辺 $\to 0$

したがって、$\Phi = \underline{\Phi_t} < +\frac{\pi}{2}$ で $l^2 = 0$

転移緯度がどこかに存在する。

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◎ 2次量の伝播:

(3-1) → (2·19) ~ (2·21) $O(\varepsilon^0)$ Note-8

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbb{F} = \frac{\jmath' F'}{\hat{\beta}} \cos\phi$$

$$\downarrow$$

$$\frac{\partial A}{\partial t} + \nabla \cdot (\mathbf{C}_g A) = \frac{\jmath' F'}{\hat{\beta}} \cos\phi \qquad (3\text{-}16)$$

or

$$\frac{\partial A}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}\left[\left(\bar{u} + \frac{k^2 - l^2}{(k^2+l^2)^2}\hat{\beta}\right)A\right] + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left[\cos\phi \cdot \frac{2kl}{(k^2+l^2)^2}\hat{\beta} A\right] = \frac{\jmath' F'}{\beta}\cos\phi \qquad (3\text{-}16)'$$

$\overline{(3\text{-}16)}$:

$$\frac{\partial \bar{A}}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left[\cos\phi \cdot C_{g\phi}\bar{A}\right] = \frac{\overline{\jmath' F'}}{\beta}\cos\phi \qquad (3\text{-}17)$$

◎ 例題:

定常強制 + Rayleigh damping

$$(3\text{-}17) \rightarrow \cancel{\frac{\partial \overline{A}}{\partial t}}^{\,\rightarrow 0} + \underline{\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi \; c_{g\phi}\overline{A}\right)} = -\frac{2}{\tau}\overline{A}$$

$$\overline{A}\cos\phi \propto e^{-2\int\frac{d\phi}{\tau c_{g\phi}}} \qquad \overline{\nabla\cdot F} \qquad (3\text{-}18)$$

$$(2\text{-}21) \rightarrow \frac{\partial}{\partial t}\overline{u}\cos\phi = \overline{\nabla\cdot F} + \overline{F_\lambda}\cos\phi$$

$$= -\frac{2}{\tau}\overline{A} + \overline{F_\lambda}\cos\phi$$

$$= -\underbrace{\boxed{\frac{2A_0}{\tau\cos\phi}e^{-2\int\frac{d\phi}{\tau c_{g\phi}}}}}_{>0} + \overline{F_\lambda}\cos\phi \qquad (3\text{-}19)$$

東風加速.

NO. Note - 1

$$\frac{D}{Dt}\{(u + a\cos\phi\ \Omega)\, a\cos\phi\}$$

$$= \frac{D}{Dt}(u\, a\cos\phi + \Omega\, a^2\cos^2\phi)$$

$$= a\cos\phi\frac{Du}{Dt} + u a\frac{D}{Dt}(\cos\phi) + \Omega a^2\frac{D}{Dt}(\cos^2\phi)$$

$$= a\cos\phi\frac{Du}{Dt} - uv\sin\phi - v\Omega a \cdot 2\cdot\cos\phi\sin\phi$$

$$= a\cos\phi\left(\frac{Du}{Dt} - \frac{uv\tan\phi}{a} - 2\Omega\sin\phi\, v\right)$$ //

$a\cos\phi \times (1-2)$

$$\frac{D}{Dt}\{(u + a\cos\phi\cdot\Omega)\, a\cos\phi\} = -\frac{1}{\rho}\frac{\partial p}{\partial\lambda} + a\cos\phi F_\lambda$$ //

flux-form

$$\frac{\partial}{\partial t}(u\, a\cos\phi) + u\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(u\, a\cos\phi) + v\frac{1}{a}\frac{\partial}{\partial\phi}\{\cdots\}$$

$$= \frac{\partial}{\partial t}(u\, a\cos\phi) + \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(u^2 a\cos\phi) - u\frac{\partial u}{\partial\lambda}$$
$$+ \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\{\cdots\}v) - \{\cdots\}\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\, v)$$

$$= \frac{\partial}{\partial t}(u\, a\cos\phi) + \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(u^2 a\cos\phi) + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\{\cdots\}v)$$
$$- u\frac{\partial u}{\partial\lambda} - (u + a\cos\phi\cdot\Omega)\frac{\partial}{\partial\phi}(\cos\phi\, v)$$

?

NO. Note·2

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + f\mathbf{e}_r\times\mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{F}$$

ただし、

$$\nabla = \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\mathbf{e}_\lambda + \frac{1}{a}\frac{\partial}{\partial\phi}\mathbf{e}_\phi \; (+\,0\,\mathbf{e}_r)$$

$$f = 2\Omega\sin\phi$$

$$(\mathbf{u}\cdot\nabla)\mathbf{u} = \nabla\frac{|\mathbf{u}|^2}{2} - \mathbf{u}\times(\nabla\times\mathbf{u})$$

$$= \nabla\frac{|\mathbf{u}|^2}{2} + (\nabla\times\mathbf{u})\times\mathbf{u}$$

$$\therefore\ \frac{\partial \mathbf{u}}{\partial t} + \nabla\frac{|\mathbf{u}|^2}{2} + (\nabla\times\mathbf{u} + f\mathbf{e}_r)\times\mathbf{u} = -\frac{1}{\underset{\text{const}}{\rho}}\nabla p + \mathbf{F}$$

$\nabla\times$

$$\frac{\partial}{\partial t}\nabla\times\mathbf{u} + (\nabla\times\mathbf{u} + f\mathbf{e}_r)\cancel{(\nabla\cdot\mathbf{u})}^{0} - \mathbf{u}\{\cancel{\nabla\cdot(\nabla\times\mathbf{u})}^{0} + \cancel{f\nabla\cdot\mathbf{e}_r}^{0}\}$$

$$- \{\underbrace{(\nabla\times\mathbf{u} + f\mathbf{e}_r)}_{/\!/\mathbf{e}_r}\cdot\nabla\}^{0}\mathbf{u} + (\mathbf{u}\cdot\nabla)(\nabla\times\mathbf{u} + f\mathbf{e}_r)$$

$$= \nabla\times\mathbf{F}$$

$$\overbrace{\left\{\frac{\partial}{\partial t} + (\mathbf{u}\cdot\nabla)\right\}}^{\frac{D}{Dt}}(\nabla\times\mathbf{u} + f\mathbf{e}_r) = \nabla\times\mathbf{F}$$

$\mathbf{e}_r\cdot$

$$\frac{D}{Dt}(\zeta + f) = \mathbf{e}_r\cdot(\nabla\times\mathbf{F})$$

$$= \frac{1}{a\cos\phi}\left[\frac{\partial F_\phi}{\partial\lambda} - \frac{\partial}{\partial\phi}(\cos\phi\, F_\lambda)\right] /\!/$$

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$$(\mathbf{u}\cdot\nabla)b = -\frac{1}{a}\frac{\partial\psi}{\partial\phi}\cdot\frac{1}{a\cos\phi}\frac{\partial b}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial\psi}{\partial\lambda}\frac{1}{a}\frac{\partial b}{\partial\phi}$$

$$= \frac{1}{a^2\cos\phi}\left(\frac{\partial\psi}{\partial\lambda}\frac{\partial b}{\partial\phi} - \frac{\partial\psi}{\partial\phi}\frac{\partial b}{\partial\lambda}\right)$$

(μ)

$$\mu = \sin\phi \qquad \frac{d\mu}{d\phi} = \cos\phi$$

$$(\mathbf{u}\cdot\nabla)b = \frac{1}{a^2}\left(\frac{\partial\psi}{\partial\lambda}\frac{\partial b}{\partial\mu} - \frac{\partial\psi}{\partial\mu}\frac{\partial b}{\partial\lambda}\right)$$

$$= \frac{1}{a^2}J(\psi, b)$$

$O(\varepsilon)$

$$\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}$$

$$\frac{\partial\zeta'}{\partial t} - \frac{1}{a^2}\frac{\partial\bar{\psi}}{\partial\mu}\frac{\partial\zeta'}{\partial\lambda} + \frac{1}{a^2}\frac{\partial\psi'}{\partial\lambda}\frac{\partial\bar{\zeta}}{\partial\mu} = F'$$

$$-\frac{a}{\cos\phi}\bar{u} \qquad a\cos\phi\, v'$$

$$\therefore \frac{\partial\zeta'}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial\zeta'}{\partial\lambda} + v'\frac{1}{a}\frac{\partial\bar{\zeta}}{\partial\phi} = F'$$

エンストロフィ

$$\zeta'\frac{\partial\zeta'}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\zeta'\frac{\partial\zeta'}{\partial\lambda} + \zeta' v'\frac{1}{a}\frac{\partial\bar{\zeta}}{\partial\phi} = \zeta F'$$

Note - 4

$O(\varepsilon^2)$

$$\underbrace{\frac{\partial \zeta^{(2)}}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial \zeta^{(2)}}{\partial \lambda} + v^{(2)}\frac{1}{a}\frac{\partial \bar{q}}{\partial \phi}}_{\sim O(\varepsilon) \text{ と同じ}}$$

$$+ \frac{1}{a^2}\left(\frac{\partial \psi'}{\partial \lambda}\frac{\partial \zeta'}{\partial \mu} - \frac{\partial \psi'}{\partial \mu}\frac{\partial \zeta'}{\partial \lambda}\right) = F^{(2)}$$

$a\cos\phi\, v'$ $\quad \frac{1}{\cos\phi}\frac{\partial}{\partial\phi}$ $\quad -\frac{a}{\cos\phi}u'$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\right)\zeta^{(2)} + v^{(2)}\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}$$

$$v'\frac{1}{a}\frac{\partial\zeta'}{\partial\phi} + u'\frac{1}{a\cos\phi}\frac{\partial\zeta'}{\partial\lambda} = F^{(2)}$$

$$\therefore \left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\right)\zeta^{(2)} + v^{(2)}\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi}$$

$$+ \left(u'\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda} + v'\frac{1}{a}\frac{\partial}{\partial\phi}\right)\zeta' = F^{(2)} \quad //$$

$$(\sim\sim \text{部}) = \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(u'\zeta') - \frac{\zeta'}{a\cos\phi}\frac{\partial u'}{\partial\lambda}$$

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\, v'\zeta') - \frac{\zeta'}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\, v')$$

$$\underbrace{\qquad}_{= -\nabla\cdot u' = 0}$$

$$= \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(\zeta' u') + \frac{\partial}{\partial\phi}(\cos\phi\, \zeta' v')\right] \quad //$$

Note-5

$$\zeta'v' = \frac{1}{a^3\cos^2\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\frac{\partial\psi'}{\partial\lambda}\right)^2\right\} + \frac{1}{a^3\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\psi'}{\partial\phi}\right)\frac{\partial\psi'}{\partial\lambda}$$

$$= \frac{1}{a^3\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\psi'}{\partial\phi}\frac{\partial\psi'}{\partial\lambda}\right) - \frac{1}{a^3\cos\phi}\frac{\partial\psi'}{\partial\phi}\frac{\partial^2\psi'}{\partial\phi\partial\lambda}$$

$$= \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\underbrace{\frac{1}{a}\frac{\partial\psi'}{\partial\phi}}_{-u'}\underbrace{\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}}_{v'}\right) - \frac{1}{a^3\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\frac{\partial\psi'}{\partial\phi}\right)^2\right\}$$

$$= \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right) - \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right)^2\right\}$$

$$= \frac{-1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left(\cos^2\phi\, u'v'\right) - \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left(\frac{1}{2}u'^2\right)$$

$$= \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{2}\left(\underbrace{\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}}_{v'}\right)^2\right\}$$

$$= \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left(\frac{1}{2}v'^2\right)$$

$$\zeta'v' = \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\lambda}\left[\left\{\frac{1}{2}\left(\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right)^2 - \frac{1}{2}\left(\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right)^2\right\}\cos\phi\right]$$

$$+ \frac{1}{a^2\cos^2\phi}\frac{\partial}{\partial\phi}\left[\cos^2\phi\;\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\;\frac{1}{a}\frac{\partial\psi'}{\partial\phi}\right]$$

$$= \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\lambda}\left\{\left(\frac{1}{2}v'^2 - \frac{1}{2}u'^2\right)\cos\phi\right\}$$

$$- \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left\{\cos^2\phi\; u'v'\right\}$$

NO. Note-6

WKB

(2-10): $\left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\right)\zeta' + v'\hat{\beta} = F'$

$$\zeta' = \Delta\psi' = \frac{1}{a^2\cos^2\phi}\frac{\partial^2\psi'}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\psi'}{\partial\phi}\right)$$

$$v' = \frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}$$

- $\hat{\beta} = \frac{1}{a}\frac{\partial\bar{q}}{\partial\phi} = \bar{\beta}(\Phi)$ slowly varying
- $\cos\phi \rightarrow \cos\Phi$ =

$$(\Lambda, \Phi, T) = \varepsilon(\lambda, \phi, t)$$

$$\left(\frac{\partial}{\partial\lambda}, \frac{\partial}{\partial\phi}, \frac{\partial}{\partial t}\right) = \varepsilon\left(\frac{\partial}{\partial\Lambda}, \frac{\partial}{\partial\Phi}, \frac{\partial}{\partial T}\right)$$

$$\frac{\partial\psi'}{\partial\lambda} = \varepsilon\frac{\partial\psi'}{\partial\Lambda} = \varepsilon\left(i\frac{1}{\varepsilon}\psi_0\frac{\partial\Theta}{\partial\Lambda} + \frac{\partial\psi_0}{\partial\Lambda} + O(\varepsilon)\right)e^{i\Theta/\varepsilon}$$

$$= \left(i\psi_0\frac{\partial\Theta}{\partial\Lambda} + \varepsilon\frac{\partial\psi_0}{\partial\Lambda} + O(\varepsilon^2)\right)e^{i\Theta/\varepsilon}$$

$$\frac{\partial^2\psi'}{\partial\lambda^2} = \varepsilon^2\frac{\partial^2\psi'}{\partial\Lambda^2} = \varepsilon\frac{\partial}{\partial\Lambda}\left\{\left(i\psi_0\frac{\partial\Theta}{\partial\Lambda} + \varepsilon\frac{\partial\psi_0}{\partial\Lambda} + O(\varepsilon^2)\right)e^{i\Theta/\varepsilon}\right\}$$

$$= \varepsilon\times\left\{i\frac{1}{\varepsilon}\frac{\partial\Theta}{\partial\Lambda}\left(i\psi_0\frac{\partial\Theta}{\partial\Lambda} + \varepsilon\frac{\partial\psi_0}{\partial\Lambda} + O(\varepsilon^2)\right)\right.$$

$$\left. + i\left(\frac{\partial\psi_0}{\partial\Lambda}\frac{\partial\Theta}{\partial\Lambda} + \psi_0\frac{\partial^2\Theta}{\partial\Lambda^2}\right) + O(\varepsilon)\right\}e^{i\Theta/\varepsilon}$$

$$= \left[-\psi_0\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \varepsilon i\left(2\frac{\partial\Theta}{\partial\Lambda}\frac{\partial\psi_0}{\partial\Lambda} + \psi_0\frac{\partial^2\Theta}{\partial\Lambda^2}\right) + O(\varepsilon^2)\right.$$

$$\times e^{i\Theta/\varepsilon}$$

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NO. Note-6+1

$$\frac{\partial \psi}{\partial \phi} = \varepsilon \frac{\partial \psi}{\partial \Phi} = \left(i\psi_0 \frac{\partial \Theta}{\partial \Phi} + \varepsilon \frac{\partial \psi_0}{\partial \Phi} + O(\varepsilon^2) \right) e^{i\Theta/\varepsilon}$$

$$\frac{\partial}{\partial \phi}\left(\cos\phi \frac{\partial \psi}{\partial \phi}\right) = \varepsilon \frac{\partial}{\partial \Phi}\left(\cos\Phi \left(\varepsilon \frac{\partial \psi}{\partial \Phi}\right)\right)$$

$$= \varepsilon \frac{\partial}{\partial \Phi} \left\{ \cos\Phi \left(i\psi_0 \frac{\partial \Theta}{\partial \Phi} + \varepsilon \frac{\partial \psi_0}{\partial \Phi} + O(\varepsilon^2) \right) e^{i\Theta/\varepsilon} \right\}$$

$$= \varepsilon \left\{ i \frac{1}{\varepsilon} \frac{\partial \Theta}{\partial \Phi} \cdot \cos\Phi \left(i\psi_0 \frac{\partial \Theta}{\partial \Phi} + \varepsilon \frac{\partial \psi_0}{\partial \Phi} + O(\varepsilon^2) \right) \right.$$

$$- \sin\Phi \left(i\psi_0 \frac{\partial \Theta}{\partial \Phi} + O(\varepsilon) \right)$$

$$\left. + \cos\Phi \left(i \frac{\partial \psi_0}{\partial \Phi} \frac{\partial \Theta}{\partial \Phi} + i\psi_0 \frac{\partial^2 \Theta}{\partial \Phi^2} + O(\varepsilon) \right) \right\} e^{i\Theta/\varepsilon}$$

$$= \left[-\cos\Phi\, \psi_0 \left(\frac{\partial \Theta}{\partial \Phi}\right)^2 + \varepsilon i \cos\Phi \left(2 \frac{\partial \psi_0}{\partial \Phi} \frac{\partial \Theta}{\partial \Phi} + \psi_0 \frac{\partial^2 \Theta}{\partial \Phi^2} \right.\right.$$

$$\left.\left. - \tan\Phi\, \psi_0 \frac{\partial \Theta}{\partial \Phi} \right) + O(\varepsilon^2) \right] e^{i\Theta/\varepsilon}$$

$$\therefore \zeta' = \left\{ \frac{-1}{a^2 \cos^2\Phi} \psi_0 \left(\frac{\partial \Theta}{\partial \Lambda}\right)^2 - \frac{1}{a^2} \psi_0 \left(\frac{\partial \Theta}{\partial \Phi}\right)^2 \right\} e^{i\Theta/\varepsilon}$$

$$+ \varepsilon i \left\{ 2 \frac{\partial \psi_0}{\partial \Lambda} \frac{\partial \Theta}{\partial \Lambda} + \psi_0 \frac{\partial^2 \Theta}{\partial \Lambda^2} + 2 \frac{\partial \psi_0}{\partial \Phi} \frac{\partial \Theta}{\partial \Phi} + \psi_0 \frac{\partial^2 \Theta}{\partial \Phi^2} \right.$$

$$\left. - \tan\Phi\, \psi_0 \frac{\partial \Theta}{\partial \Phi} \right\} e^{i\Theta/\varepsilon}$$

$$+ O(\varepsilon^2)$$

$$\frac{\partial \zeta'}{\partial t} = \varepsilon \frac{\partial \zeta'}{\partial T} = -i \frac{\partial \Theta}{\partial T} \left\{ \frac{1}{a^2 \cos^2\Phi} \left(\frac{\partial \Theta}{\partial \Lambda}\right)^2 + \frac{1}{a^2} \left(\frac{\partial \Theta}{\partial \Phi}\right)^2 \right\} \psi_0\, e^{i\Theta/\varepsilon}$$

$$- \varepsilon \frac{\partial}{\partial T} \left[\left\{ \frac{1}{a^2 \cos^2\Phi} \left(\frac{\partial \Theta}{\partial \Lambda}\right)^2 + \frac{1}{a^2} \left(\frac{\partial \Theta}{\partial \Phi}\right)^2 \right\} \psi_0 \right] e^{i\Theta/\varepsilon}$$

$$+ \varepsilon i^2 \frac{\partial \Theta}{\partial T} \left[2 \left(\frac{\partial \psi_0}{\partial \Lambda} \frac{\partial \Theta}{\partial \Lambda} + \frac{\partial \psi_0}{\partial \Phi} \frac{\partial \Theta}{\partial \Phi} \right) + \right.$$

$$\left. \left\{ \frac{\partial^2 \Theta}{\partial \Lambda^2} + \frac{1}{\cos\Phi} \frac{\partial}{\partial \Phi} \left(\cos\Phi \frac{\partial \Theta}{\partial \Phi} \right) \right\} \psi_0 \right] e^{i\Theta/\varepsilon}$$

$$\frac{1}{\cos\phi} \frac{\partial}{\partial \phi} \left(\cos\phi \frac{\partial \Theta}{\partial \Phi} \right) + O(\varepsilon^2)$$

$\dfrac{\bar{u}}{a\cos\Phi}\dfrac{\partial}{\partial\lambda}$ も同様

$$v'\hat{\beta} = \frac{1}{a\cos\Phi}\frac{\partial\psi'}{\partial\lambda}\hat{\beta} = \frac{\hat{\beta}}{a\cos\Phi}\left(i\psi_0\frac{\partial\Theta}{\partial\Lambda} + \varepsilon\frac{\partial\psi_0}{\partial\Lambda} + O(\varepsilon^2)\right)e^{i\Theta/\varepsilon}$$

$\boxed{O(\varepsilon^0)}$:

$$-i\left(\frac{\partial\Theta}{\partial T} + \frac{\bar{u}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\right)\left\{\frac{1}{a^2\cos^2\Phi}\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \frac{1}{a^2}\left(\frac{\partial\Theta}{\partial\Phi}\right)^2\right\}\psi_0 e^{i\Theta/\varepsilon}$$
$$+ \frac{i}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\hat{\beta}\psi_0 e^{i\Theta/\varepsilon} = 0$$

$\div\ -i\psi_0 e^{i\Theta/\varepsilon}$

$$\left(\frac{\partial\Theta}{\partial T} + \frac{\bar{u}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\right)\left\{\frac{1}{a^2\cos^2\Phi}\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \frac{1}{a^2}\left(\frac{\partial\Theta}{\partial\Phi}\right)^2\right\}$$
$$- \frac{\hat{\beta}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda} = 0$$

$\boxed{O(\varepsilon)}$

$$-\left(\frac{\partial}{\partial T} + \frac{\bar{u}}{a\cos\Phi}\frac{\partial}{\partial\Lambda}\right)\left[\left\{\frac{1}{a^2\cos^2\Phi}\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \frac{1}{a^2}\left(\frac{\partial\Theta}{\partial\Phi}\right)^2\right\}\psi_0\right]e^{i\Theta/\varepsilon}$$
$$-\left(\frac{\partial\Theta}{\partial T} + \frac{\bar{u}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\right)\left[2\left(\frac{\partial\psi_0}{\partial\Lambda}\frac{\partial\Theta}{\partial\Lambda} + \frac{\partial\psi_0}{\partial\Phi}\frac{\partial\Theta}{\partial\Phi}\right)\right.$$
$$\left. + \left\{\frac{\partial^2\Theta}{\partial\Lambda^2} + \frac{1}{\cos\Phi}\frac{\partial}{\partial\Phi}\left(\cos\Phi\frac{\partial\Theta}{\partial\Phi}\right)\right\}\psi_0\right]e^{i\Theta/\varepsilon}$$
$$+ \frac{\hat{\beta}}{a\cos\Phi}\frac{\partial\psi_0}{\partial\Lambda}e^{i\Theta/\varepsilon} = 0$$

$\boxed{\text{OLD}}$

NO. Note 6

$$(2\text{-}10):\ \left(\frac{\partial}{\partial t} + \bar{u}\frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}\right)\zeta' + v'\underbrace{\frac{1}{a}\frac{\partial \bar{q}}{\partial \phi}}_{\hat{\beta}} = F' \to 0$$

$$\frac{\partial}{\partial t} = \varepsilon\frac{\partial}{\partial T}, \quad \frac{\partial}{\partial \lambda} = \varepsilon\frac{\partial}{\partial \Lambda}, \quad \frac{\partial}{\partial \phi} = \varepsilon\frac{\partial}{\partial \Phi}$$

$$\zeta' = \Delta\psi' = \frac{1}{a^2\cos^2\phi}\frac{\partial^2\psi'}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\psi'}{\partial\phi}\right)$$

$$v' = \frac{1}{a\cos\phi}\frac{\partial\psi}{\partial\lambda}$$

- $\frac{1}{a}\frac{\partial\bar{q}}{\partial\phi} = \hat{\beta}(\Phi)$

- $\cos\phi \longrightarrow \cos\Phi$ slowly varying

$$\zeta' = \Delta\psi = -\left\{\frac{1}{a^2\cos^2\Phi}\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \frac{1}{a^2}\left(\frac{\partial\Theta}{\partial\Phi}\right)^2\right\}\psi_0 e^{i\Theta/\varepsilon} + O(\varepsilon)$$

(局所波数 $k = \frac{1}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}$, $l = \frac{1}{a}\frac{\partial\Theta}{\partial\Phi}$)

$O(\varepsilon^0)$

$$\left[i\Big(\underbrace{\frac{\partial\Theta}{\partial T}}_{-\omega} + \frac{\bar{u}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\Big)\times -\left\{\frac{1}{a^2\cos^2\Phi}\left(\frac{\partial\Theta}{\partial\Lambda}\right)^2 + \frac{1}{a^2}\left(\frac{\partial\Theta}{\partial\Phi}\right)^2\right\} + \frac{i\tilde{\beta}}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}\right]\psi_0 e^{i\Theta/\varepsilon} = 0 \quad //$$

$$(-\omega + \bar{u}k)(k^2 + l^2) = k\hat{\beta}$$

$$\omega = \bar{u}k - \frac{k\hat{\beta}}{k^2+l^2} \quad //$$

NO. Note-7

$$C_{g\lambda} = \frac{\partial \omega}{\partial (k\cos\Phi)} \cdot \cos\Phi$$

$$= \bar{u} - \frac{\frac{\hat{\beta}}{\cos\Phi}\left(\frac{(k\cos\Phi)^2}{\cos^2\Phi} + l^2\right) - (k\cos\Phi)\frac{\hat{\beta}}{\cos\Phi}\cdot\frac{2(k\cos\Phi)}{\cos^2\Phi}}{\left(\frac{(k\cos\Phi)^2}{\cos^2\Phi} + l^2\right)^2} \times\cos\Phi$$

$$= \bar{u} - \frac{\hat{\beta}(k^2+l^2) - 2k^2\hat{\beta}}{(k^2+l^2)^2}$$

$$= \bar{u} - \frac{\hat{\beta}(l^2-k^2)}{(k^2+l^2)^2}$$

$$= \bar{u} + \frac{\hat{\beta}(k^2-l^2)}{(k^2+l^2)^2}$$

$$= \bar{u} + \hat{\omega}^2 \cdot \frac{1}{\hat{\beta}k^2}(k^2-l^2) \quad //$$

$$C_{g\phi} = \frac{\partial \omega}{\partial l}$$

$$= \frac{\hat{\beta}\, 2kl}{(k^2+l^2)^2}$$

$$= \hat{\omega}^2 \cdot \frac{1}{\hat{\beta}}\frac{2l}{k} \quad //$$

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(2-20):

$$A = \frac{1}{\hat\beta}\frac{\zeta'^2}{2}\cos\Phi$$

$$= \frac{\cos\Phi}{2\hat\beta}\left\{-(k^2+l^2)\psi_0 e^{i\Theta/\varepsilon}\right\}^2 + O(\varepsilon)$$

$$= \frac{\cos\Phi}{2\hat\beta}(k^2+l^2)^2\psi_0^2 e^{i2\Theta/\varepsilon} + O(\varepsilon)$$

(2-21):

$$\nabla\cdot F = \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left[A\bar u + \frac{1}{2}\left\{\left(\frac{1}{a\cos\phi}\frac{\partial\psi'}{\partial\lambda}\right)^2 - \left(\frac{1}{a}\frac{\partial\psi}{\partial\phi}\right)^2\right\}\cos\phi\right]$$
$$+\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left[\cos\phi\cdot u'v'\cos\phi\right]$$

$$\frac{1}{2}\left\{\left(\frac{1}{a\cos\phi}\frac{\partial\psi}{\partial\lambda}\right)^2 - \left(\frac{1}{a}\frac{\partial\psi}{\partial\phi}\right)^2\right\}\cos\phi$$

$$= \frac{1}{2}\left\{\Big(\underbrace{\frac{i}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}}_{k}\psi_0 e^{i\Theta/\varepsilon}\Big)^2 - \Big(\underbrace{\frac{i}{a}\frac{\partial\Theta}{\partial\Phi}}_{l}\psi_0 e^{i\Theta/\varepsilon}\Big)^2 + O(\varepsilon)\right\}\cos\Phi$$

$$= \frac{1}{2}(k^2-l^2)\psi_0^2 e^{i2\Theta/\varepsilon}\cos\Phi + O(\varepsilon)$$

$$= \frac{k^2-l^2}{(k^2+l^2)^2}\hat\beta A + O(\varepsilon)$$

$u'v'\cos\phi$

$$= \left\{-\underbrace{\frac{i}{a}\frac{\partial\Theta}{\partial\Phi}}_{l}\psi_0 e^{i\Theta/\varepsilon} + O(\varepsilon)\right\}\left\{\underbrace{\frac{i}{a\cos\Phi}\frac{\partial\Theta}{\partial\Lambda}}_{k}\psi_0 e^{i\Theta/\varepsilon} + O(\varepsilon)\right\}\times\cos\Phi$$

$$= kl\,\psi_0^2 e^{i2\Theta/\varepsilon}\cos\Phi + O(\varepsilon)$$

$$= \frac{2kl}{(k^2+l^2)^2}\hat{\beta} A \quad + O(\varepsilon)$$

(2-19)

$$\frac{\partial A}{\partial t} + \nabla \cdot F = \frac{S' F'}{\hat{\beta}} \cos\Phi$$

$C_{g\lambda}$ (3-7)

$$\frac{\partial A}{\partial t} + \frac{1}{a\cos\Phi}\frac{\partial}{\partial \lambda}\left[\left(\bar{u} + \frac{k^2-l^2}{(k^2+l^2)^2}\hat{\beta}\right) A\right]$$

$$+ \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left[\cos\phi \; \frac{2kl}{(k^2+l^2)^2}\hat{\beta} A\right]$$

$C_{g\phi}$ (3-8)

$$= \frac{S' F'}{\hat{\beta}}\cos\phi$$

$$\therefore \frac{\partial A}{\partial t} + \nabla \cdot (C_g A) = \frac{S' F'}{\hat{\beta}}\cos\phi$$

$$\frac{\partial \bar{A}}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi \; C_{g\phi}\bar{A}\right) = \frac{\overline{S' F'}}{\hat{\beta}}\cos\phi$$

(林 1-↑ 92·93)

球面2次元非発散方程式

$$\left(\frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi}\right)u - \frac{\tan\phi\, uv}{a} - 2\Omega\sin\phi\, v = -\frac{1}{\rho_0}\frac{1}{a\cos\phi}\frac{\partial p}{\partial\lambda} + f_\lambda \quad \cdots (1)$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi}\right)v + \frac{\tan\phi\, u^2}{a} + 2\Omega\sin\phi\, u = -\frac{1}{\rho_0}\frac{1}{a}\frac{\partial p}{\partial\phi} + f_\phi \quad \cdots (2)$$

$$\frac{1}{\cos\phi}\frac{\partial u}{\partial\lambda} + \frac{1}{\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\, v) = 0 \quad \cdots (3)$$

$(u, v, p ;\ f_\lambda, f_\phi)$

角運動量保存則

(1) →

$$\left(\frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi}\right)\{\overbrace{a\cos\phi(u + a\cos\phi\,\Omega)}^{M}\} = -\frac{1}{\rho_0}\frac{\partial p}{\partial\lambda} + a\cos\phi\, f_\lambda \quad \cdots (4)$$

flux form :

$$\frac{\partial M}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(uM) + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(vM\cos\phi) = -\frac{1}{\rho_0}\frac{\partial p}{\partial\lambda} + a\cos\phi\, f_\lambda \quad \cdots (5)$$

vector invariant form :

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial\lambda}\frac{u^2+v^2}{2} - (2\Omega\sin\phi + \zeta)\, v a\cos\phi = \frac{1}{\rho_0}\frac{\partial p}{\partial\lambda} + a\cos\phi\, f_\lambda \quad \cdots (6)$$

zonal mean: $\overline{} = \frac{1}{2\pi}\int_0^{2\pi} d\lambda$

$\overline{(6)}$

$$\frac{\partial}{\partial t}(a\cos\phi\,\bar{u}) \underbrace{- \overline{v\zeta}\,a\cos\phi}_{\text{角運動量流束の収束}} = a\cos\phi\,\overline{F_\lambda} \quad \cdots(7)$$

$$-\overline{v\zeta}\,a\cos\phi = \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{vM}\cos\phi)$$

$$\left(\frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial \lambda} + \frac{v}{a}\frac{\partial}{\partial \phi}\right)\{a\cos\phi(u + a\cos\phi\,\Omega)\}$$

$$= a\cos\phi\left(\frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial \lambda} + \frac{v}{a}\frac{\partial}{\partial \phi}\right)u - uv\sin\phi - a v\Omega\, 2\cos\phi\sin\phi$$

$$= a\cos\phi\left\{\left(\frac{\partial}{\partial t}\cdots\right)u - \frac{uv\tan\phi}{a} - 2\Omega v\sin\phi\right\} \quad \rightarrow (4)$$

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(4) 左辺 $+ \frac{1}{a}M \times (3)$

$$\frac{\partial M}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial M}{\partial \lambda} + \frac{M}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{v}{a}\frac{\partial M}{\partial \phi} + \frac{M}{a\cos\phi}\frac{\partial}{\partial \phi}(v\cos\phi)$$

重

$$= \frac{\partial M}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}(uM) + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(vM\cos\phi) \quad \rightarrow (5)$$

左辺