

Equatorial waves and superrotation in the stratosphere of a Titan general circulation model

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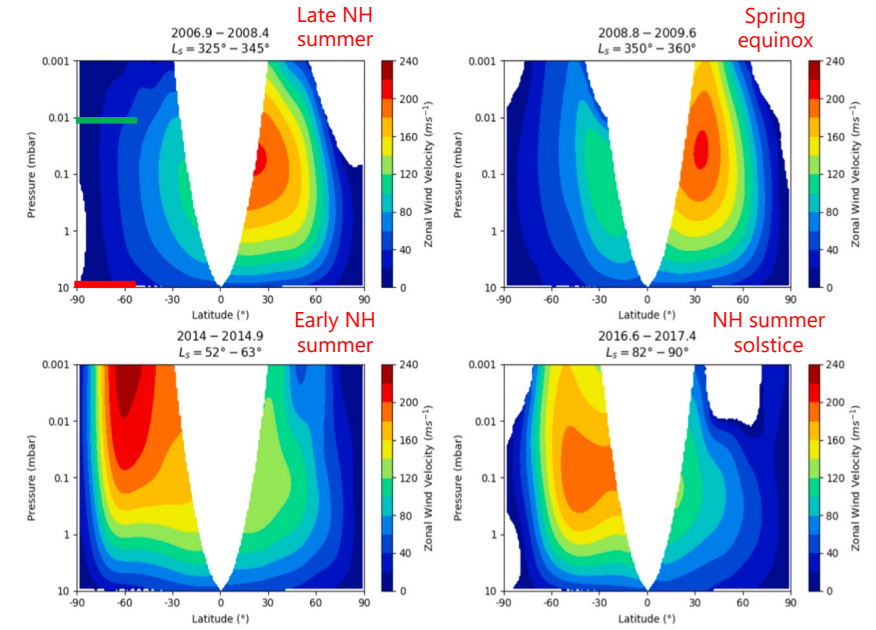
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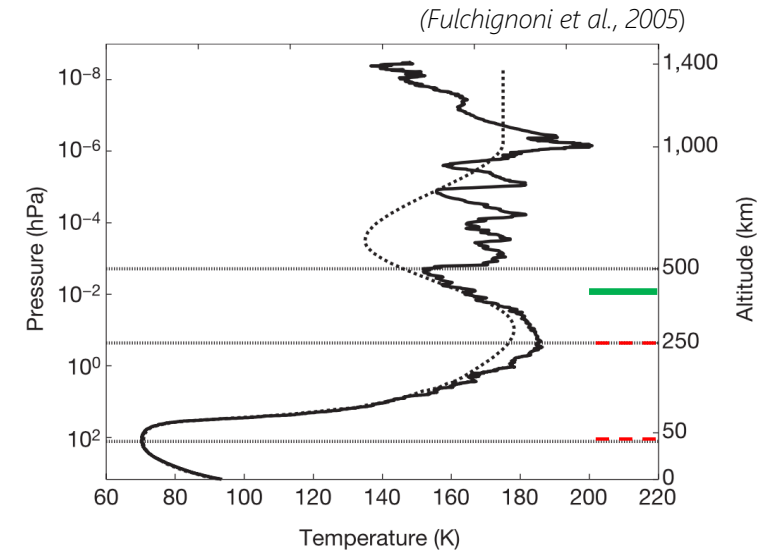
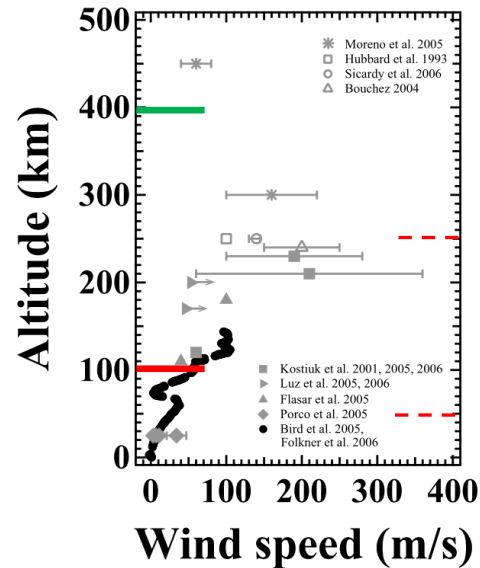
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1 Key features of Titan

Parameter	Titan	Earth
★ Radius (km)	2575	6371
★ Rotation period (Earth days)	16	1
★ Surface gravity (m s ⁻²)	1.35	9.81
★ Surface pressure (bar)	1.5	1
Atmospheric composition	94% N ₂ 5% CH ₄	78% N ₂ 21% O ₂ ~.1% H ₂ O
★ Insolation (W m ⁻²)	14.9	1368
★ Obliquity (deg)	27	23.45
Eccentricity	0.029	0.0167
★ Length of year (Earth years)	29	1

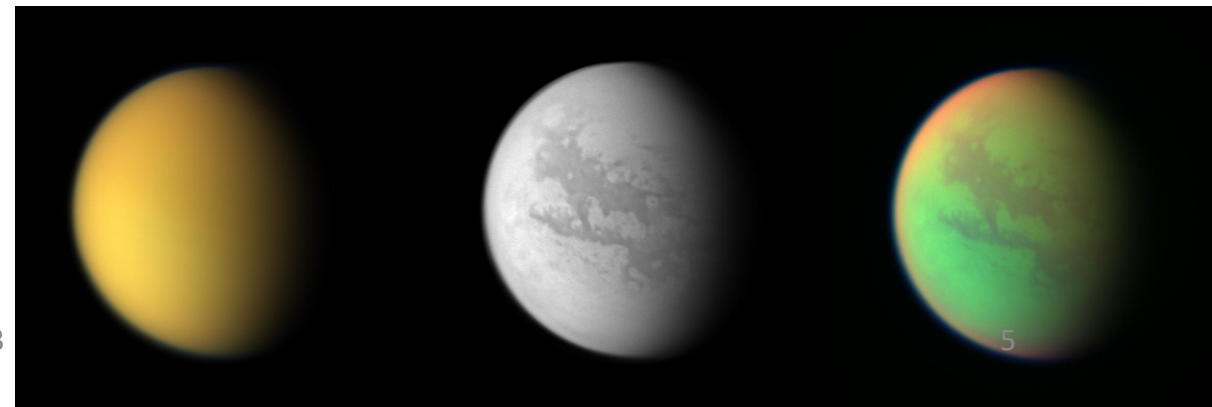
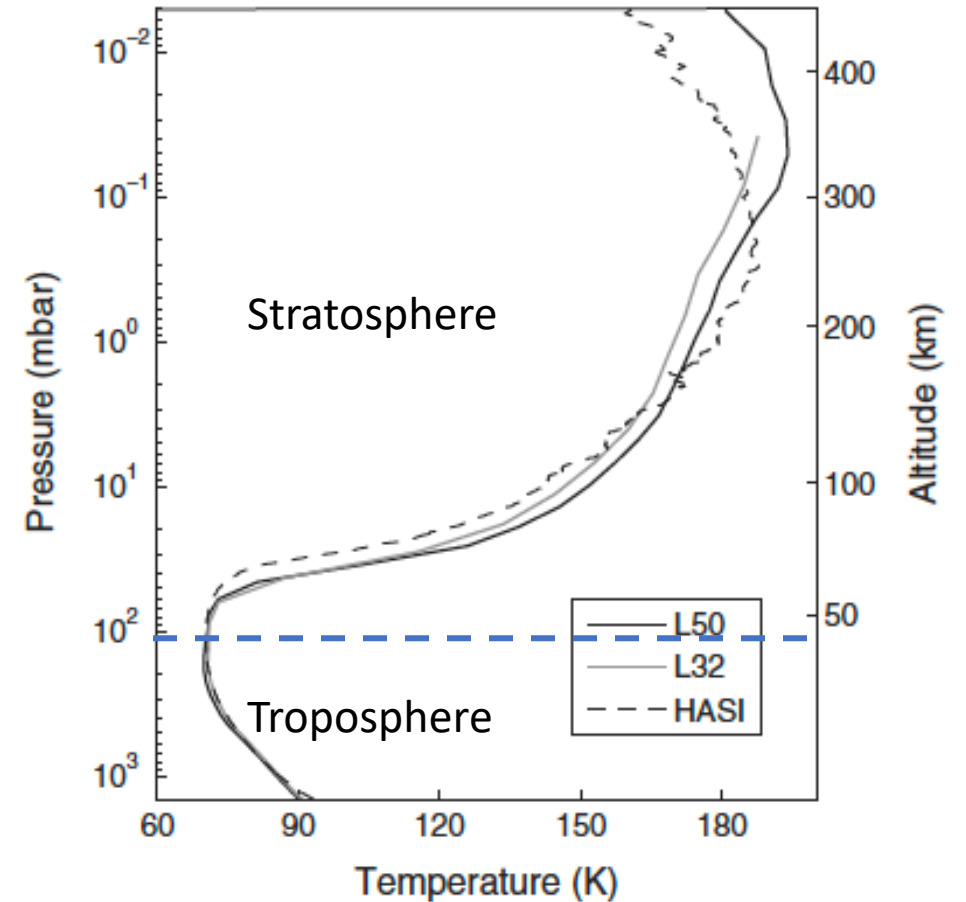


(Hörst, 2017)



Key features of Titan

- Complicated radiative budget and T profile
 - Greenhouse-like troposphere
 - Heated via surface
 - Anti-greenhouse stratosphere
 - Heated directly via hazes
- Complicated chemistry and thermodynamics
 - Optically thick photochemical hazes in stratosphere
 - Condensible constituents (CH_4) with clouds, rain, lakes....
- Strong seasonal variability



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2 Equatorial superrotation – what is it?

Super-rotation is a phenomenon in atmospheric dynamics where the axial angular momentum of an atmosphere in-some-way exceeds that of the underlying planet.

A maximum off of the equator will typically be inertially unstable, so super-rotation is invariably manifest as a zonal jet centred on the equator.

Hide's theorem says that in axisymmetric, inviscid atmosphere

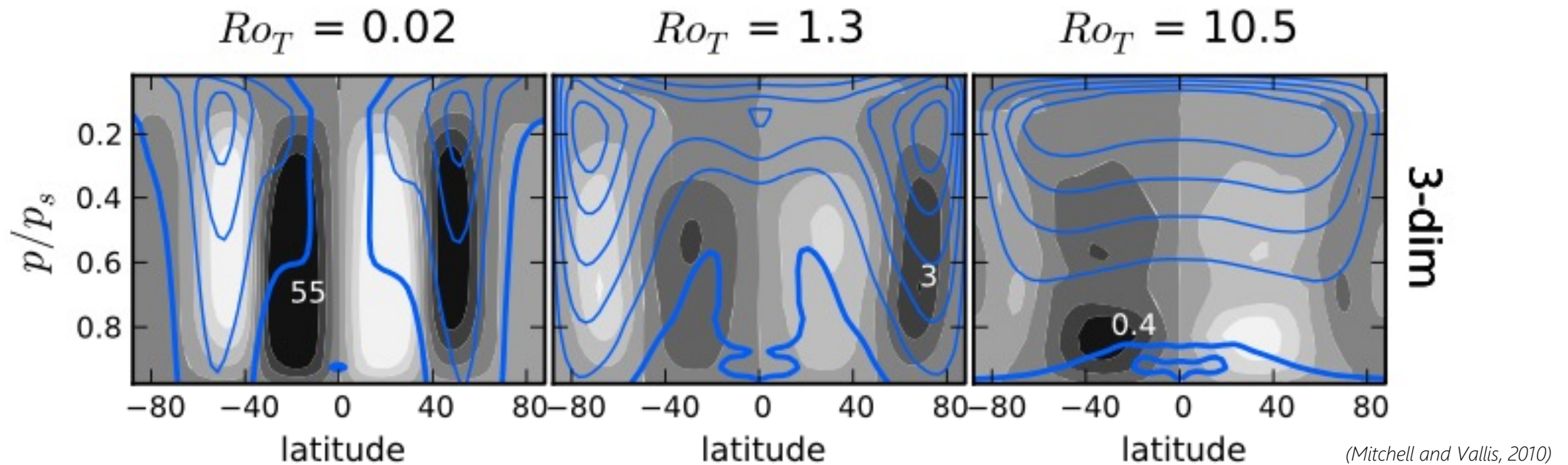
$$\frac{D\overline{m}}{Dt} = 0$$

where $m = a \cos \vartheta (\Omega a \cos \vartheta + u)$, and the overline denotes a zonal average.

Friction + overturning will lead to $m = 0$. Therefore, some process that can transport m up-gradient is required. This can be achieved by eddies.

2 Equatorial superrotation – results from idealised modeling

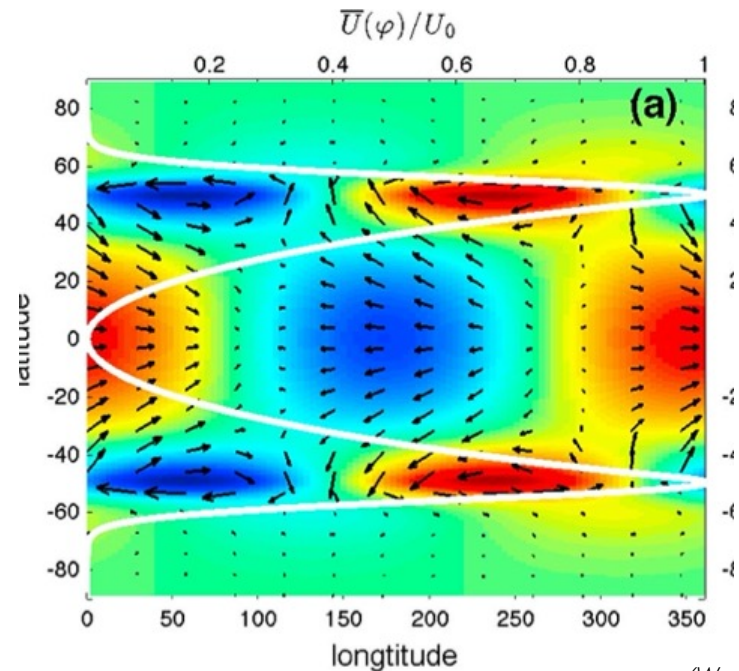
Idealised 'Earth-like' models show that superrotation emerges naturally for planets with low rotation rate / radius.



2 Equatorial superrotation – results from idealised modeling

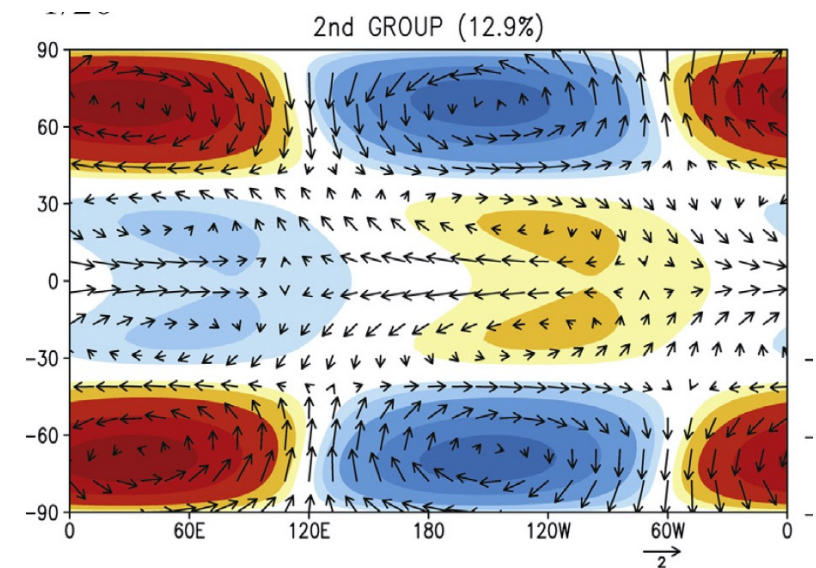
Idealised 'Earth-like' models show that superrotation emerges naturally for planets with low rotation rate / radius.

Barotropic, ageostrophic instability generates a 'Rossby—Kelvin wave' that induces equatorward momentum flux *during spin-up*:



(Wang and Mitchell, 2014)

Superrotation maintained by 'equatorial Rossby waves', generated by baroclinic/barotropic instability:

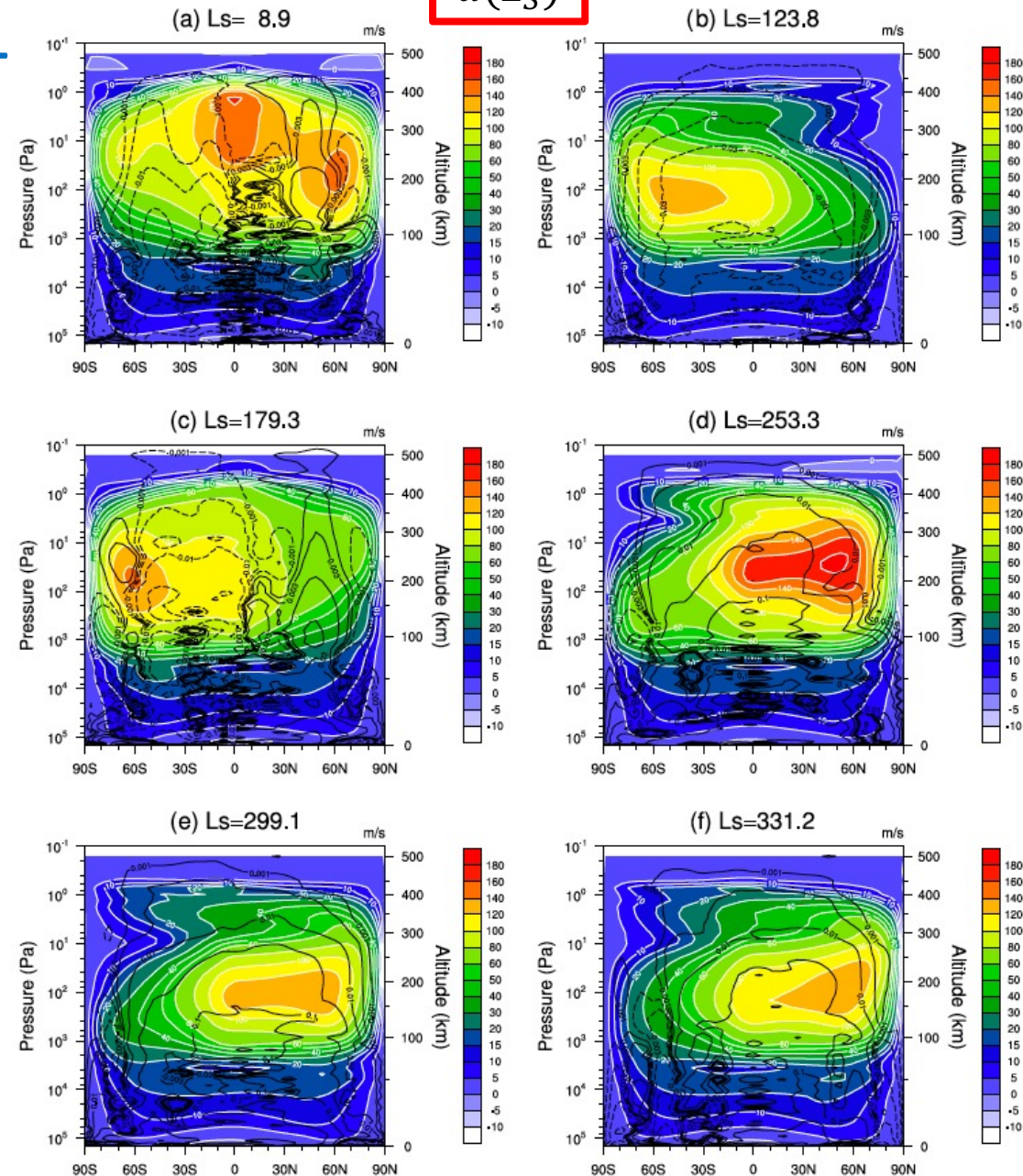


(Dias-Pinto and Mitchell, 2014)

Equatorial super-rotation – what about Titan?

- Situation on Titan is complicated by seasonal cycle and latitudinally asymmetric shear

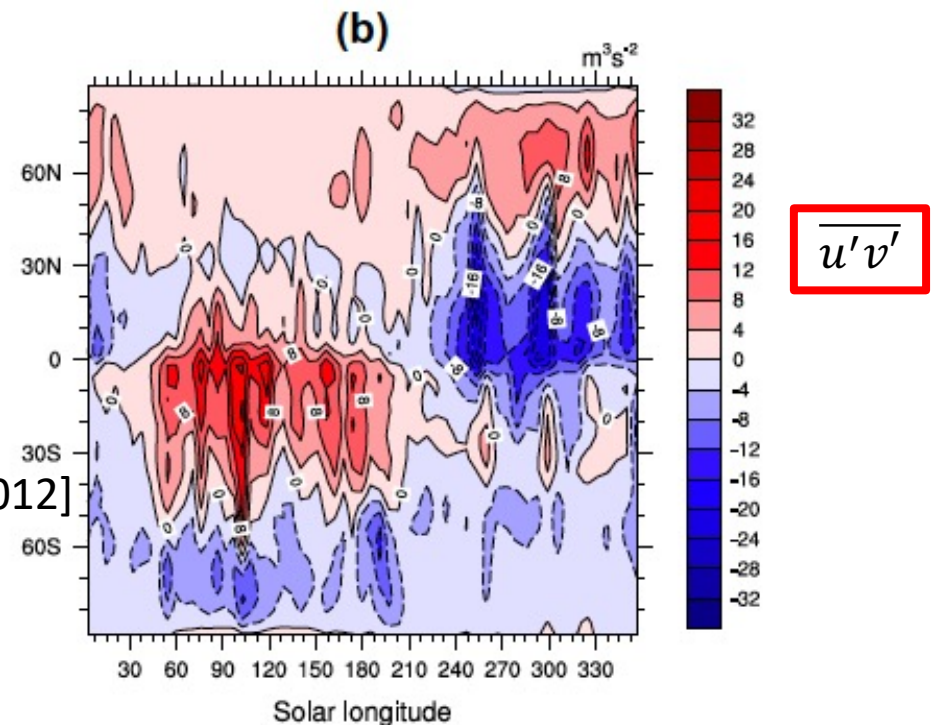
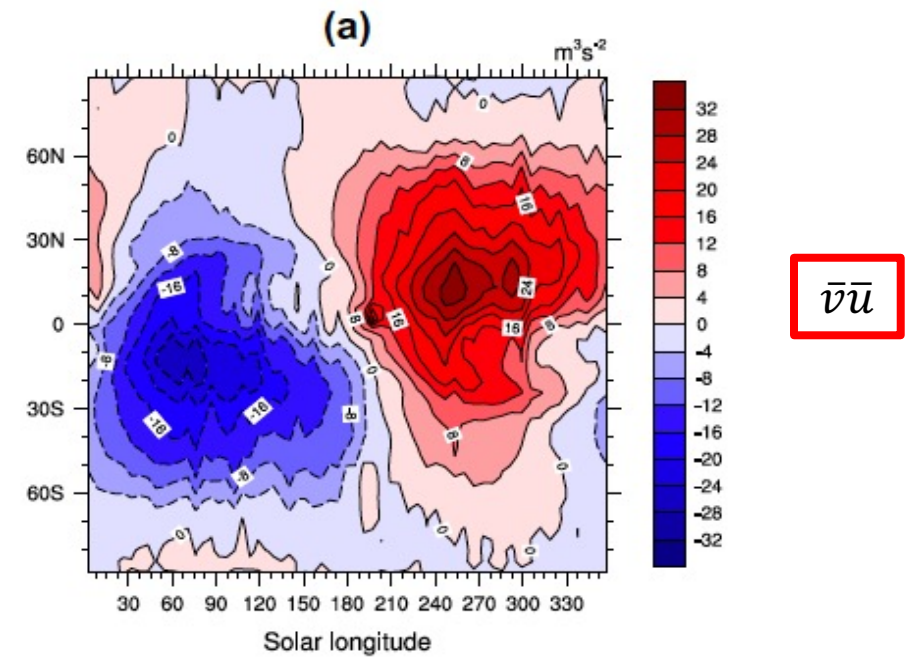
$$\bar{u}(L_S)$$



From Lebonnois et al. [2012]

Equatorial super-rotation – what about Titan?

- Situation on Titan is complicated by seasonal cycle and latitudinally asymmetric shear
- Leads to a seasonal cycle in the momentum budget



From Lebonnois et al. [2012]

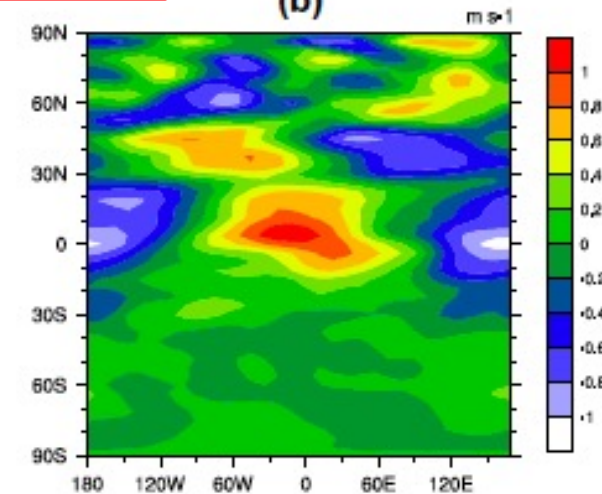
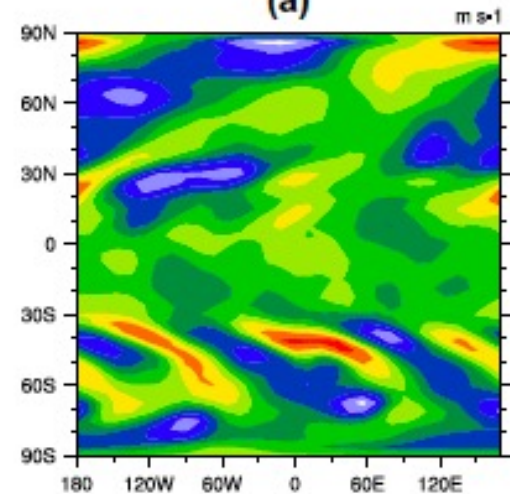
Equatorial super-rotation – what about Titan?

- Situation on Titan is complicated by seasonal cycle and latitudinally asymmetric shear
- Leads to a seasonal cycle in the momentum budget
- What are the eddies like?
 - Which contribute most to AM transport?
 - where do they come from?
 - Regular barotropic instability?

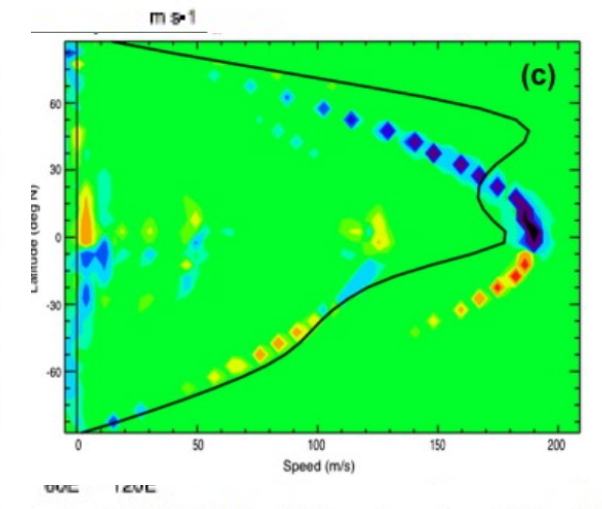
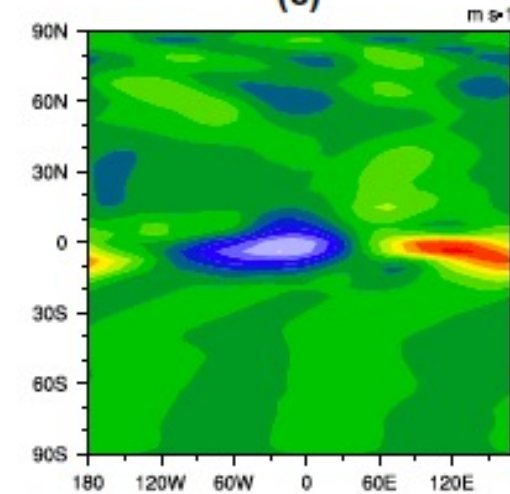
Lower troposphere
 $\omega < 4$ cycles/T day

$$u'(\omega_n)$$

Stratosphere
 ~ 6 cycles/T day



Stratosphere: ~ 11 cycles/T day.

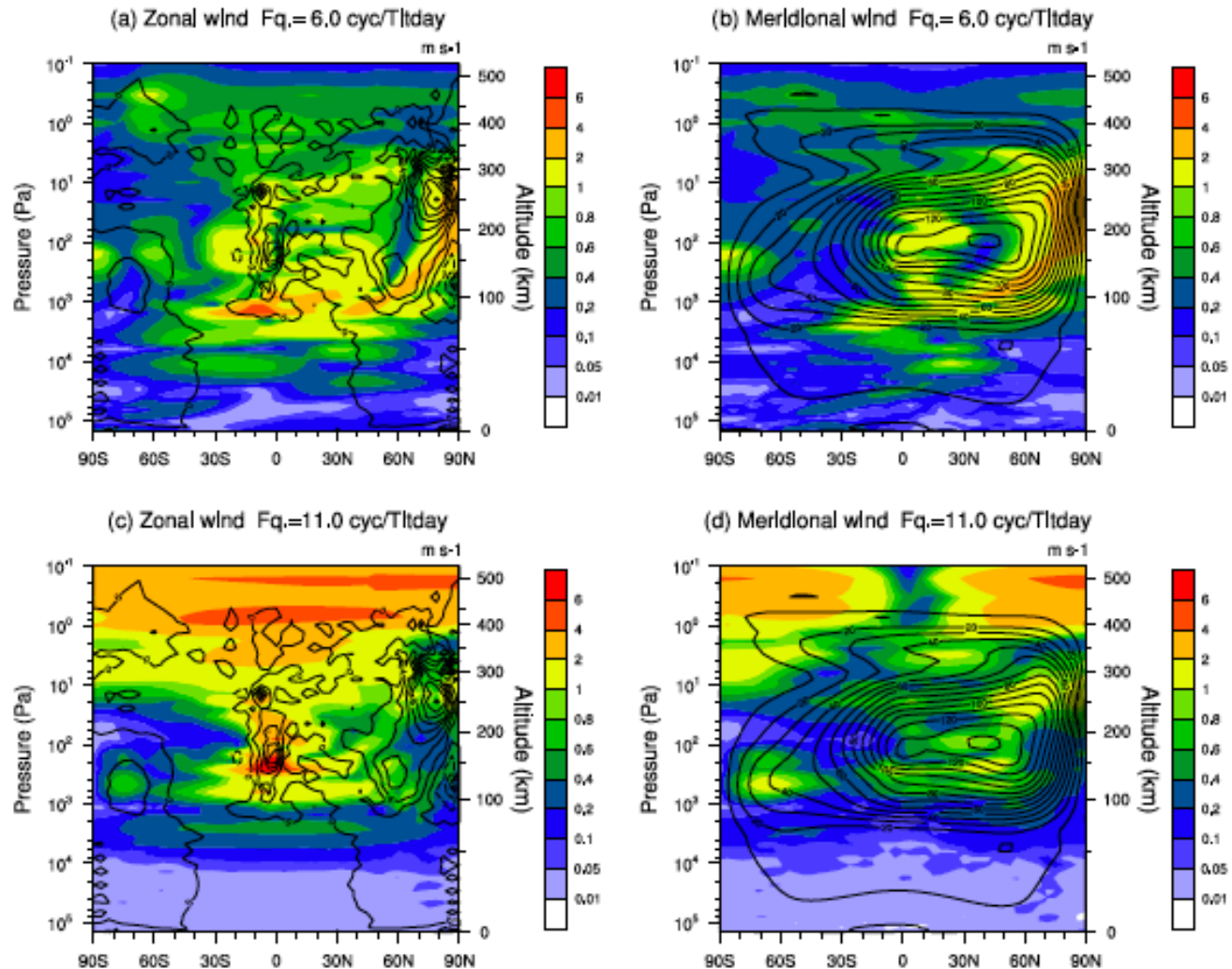


From Lebonnois et al. [2012]

Newman et al. [2011]

Equatorial super-rotation – what about Titan?

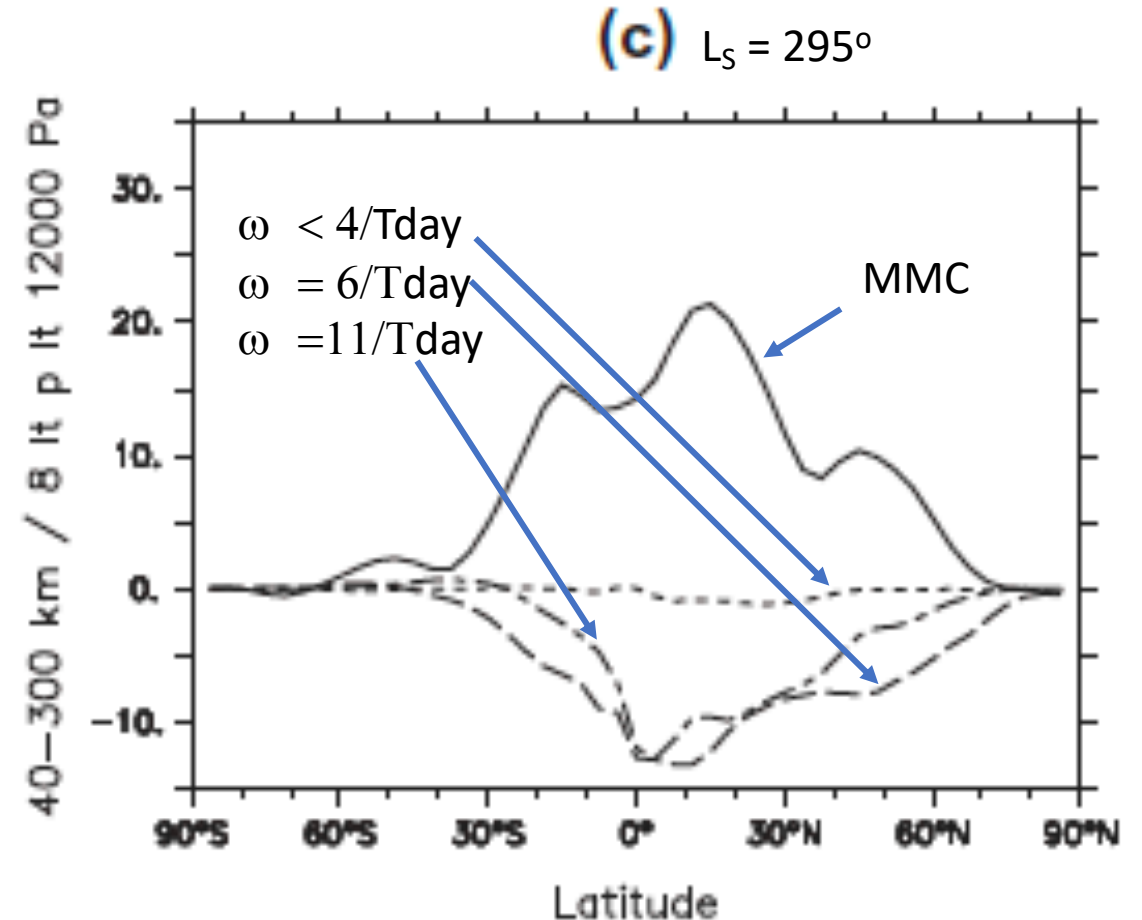
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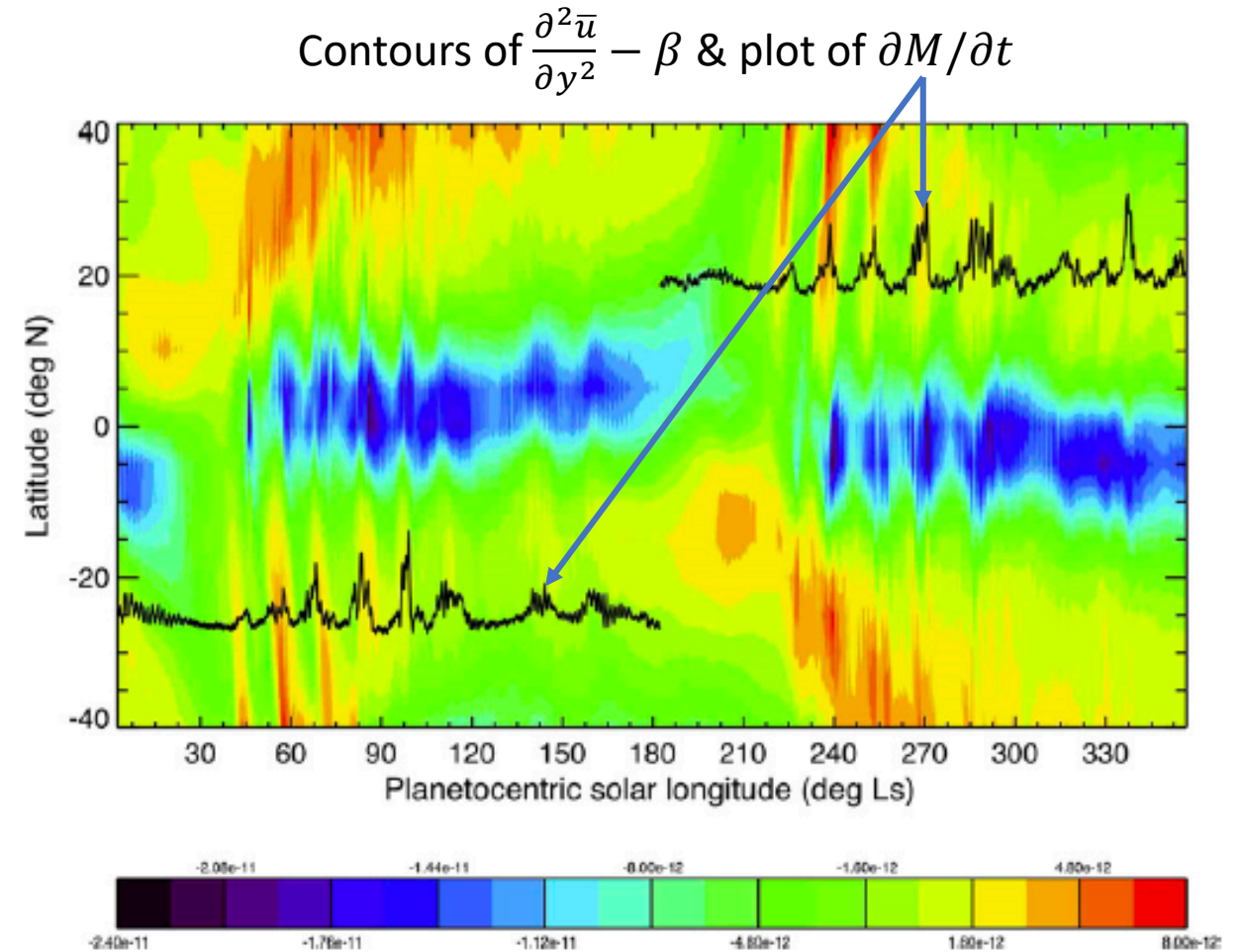
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From Newman et al. [2011]

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3 Open questions + aim

Which (equatorial) wave modes are supported in Titan's stratosphere, and what is their spatial structure?

Which contribute to maintaining equatorial superrotation?

What generates them?

What relevance do the results from idealised modelling have?

- Theoretical understanding
- Observed behaviour of Titan's climate?

Our aim: to answer some of these!

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4 Model configuration

Analyse equatorial waves and eddy angular momentum fluxes in TAM

(Lombardo and Lora 2023)

TAM = Titan Atmospheric Model

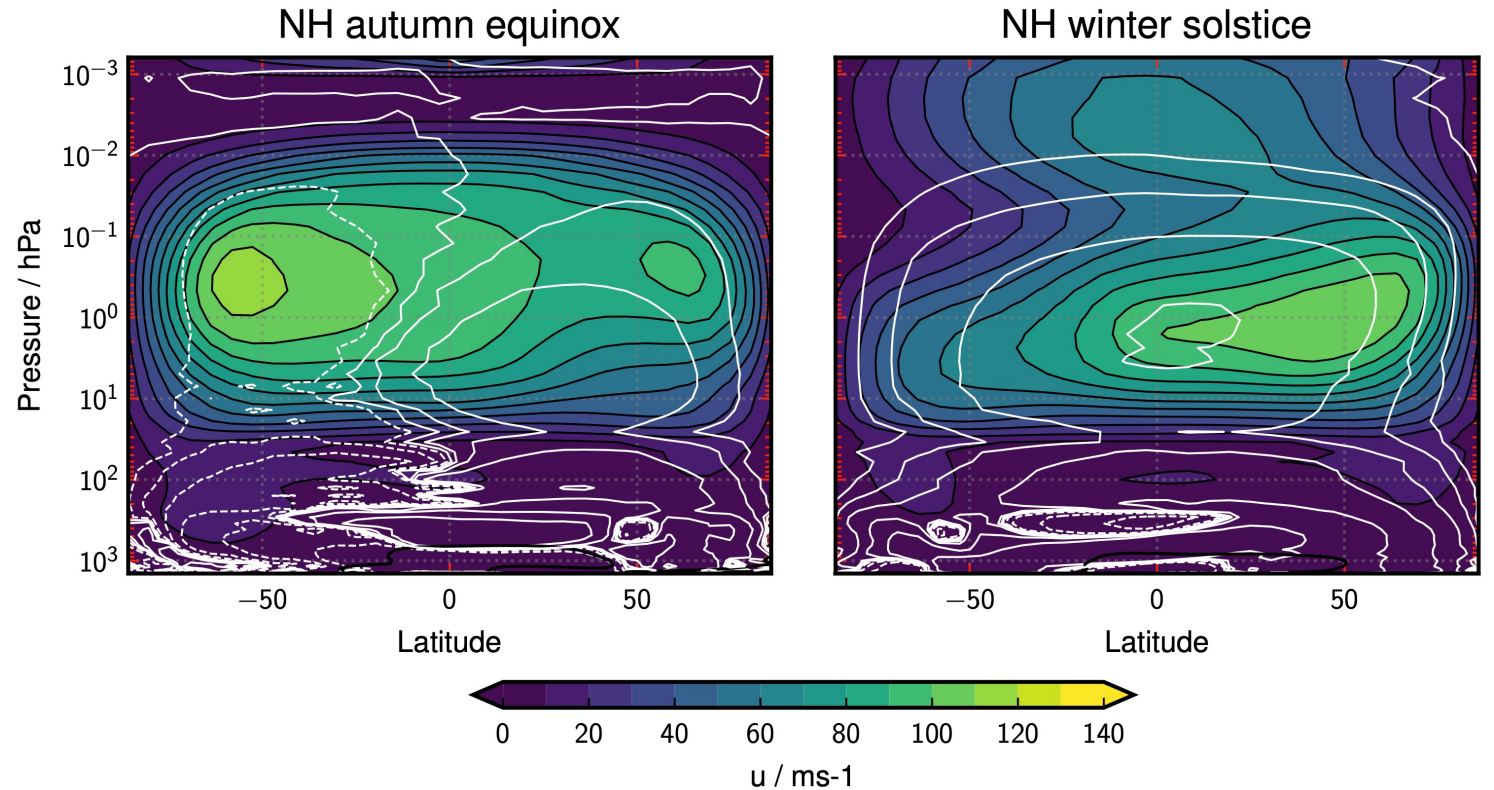
GFDL spectral core with T21 horizontal resolution (~4.5 degree)

50 levels in vertical 0 to ~450km

Realistic RT with seasonal cycle and absorption of solar radiation due to haze

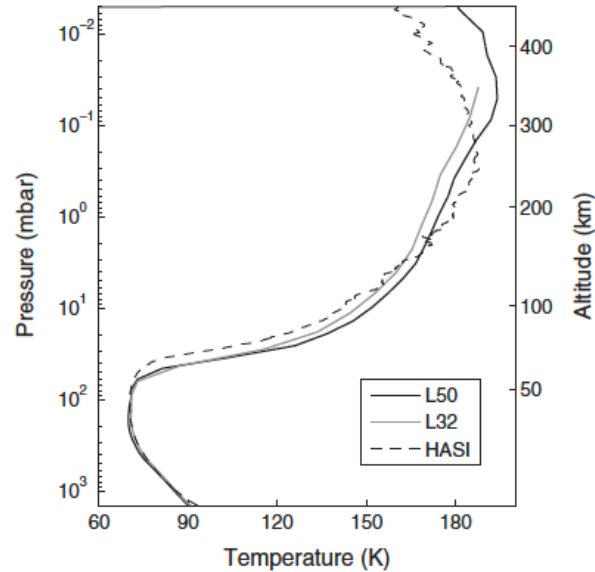
Methane hydrological cycle

Data coverage from L_{sol} 180° – 360°, output is 6hrly average (NH autumn equinox -> NH winter -> NH spring equinox)

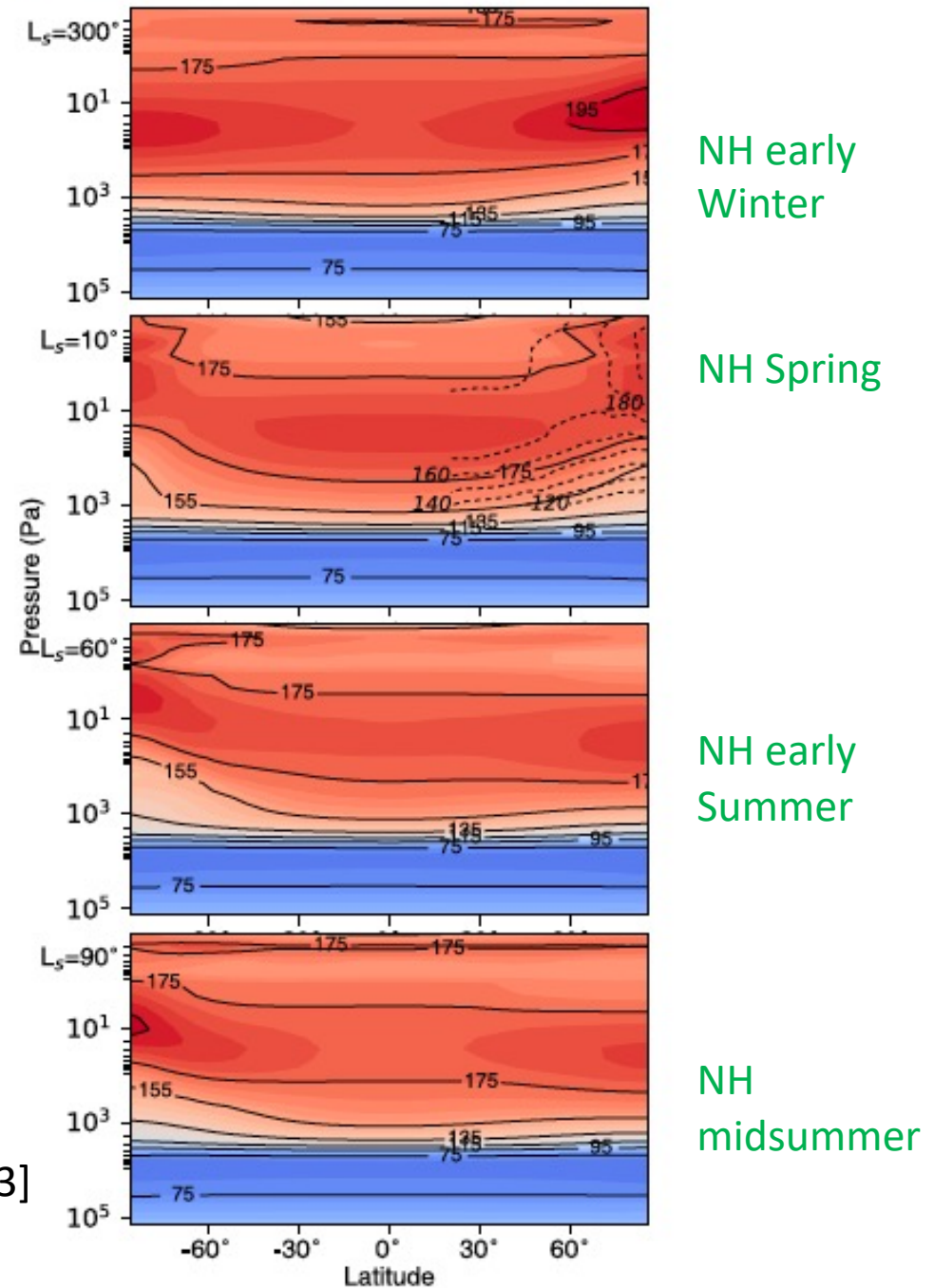


TAM Model configuration

- GFDL spectral dynamical core
 - Cf ISCA model
- T21 horizontal resolution
 - $\sim 4.5^\circ$ resolution in lat x lon
- 50 vertical levels
 - $0 < z < 450$ km
- Realistic radiative transfer
 - seasonal cycle,
 - solar absorption due to haze
 - IR cooling due to hydrocarbons and CIA due to N_2 etc.
- Mellor-Yamada boundary layer
- Betts-Miller convection
- CH_4 hydrological cycle
- Variable chemical composition [potentially]



Lombardo & Lora [2023]



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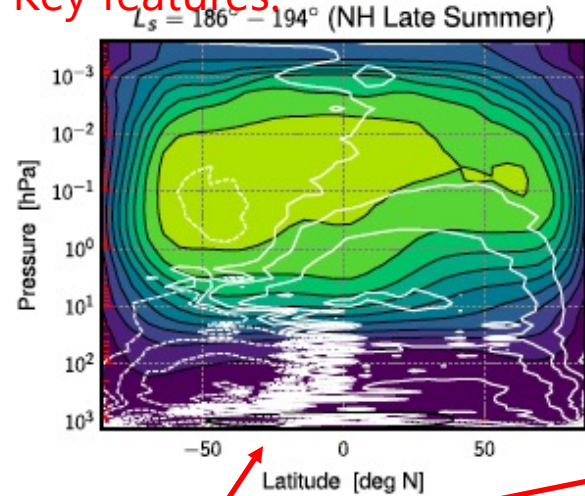
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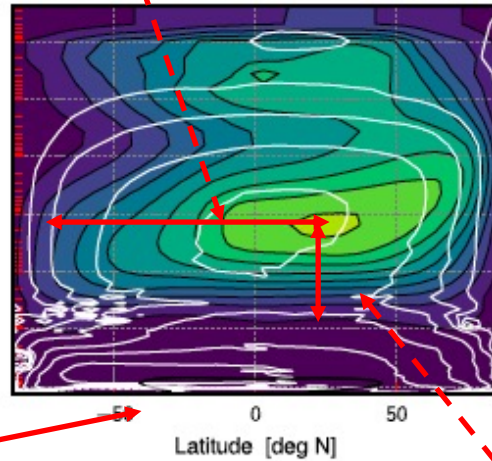
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5 Basic circulation and eddy momentum fluxes

Key features:



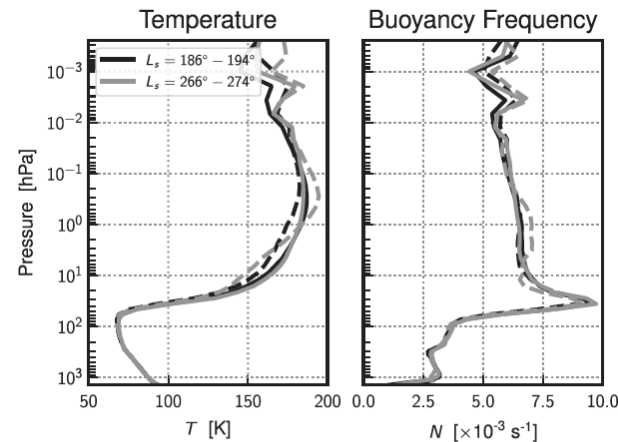
Meridional shear
 $L_s = 266^\circ - 274^\circ$ (NH Late Autumn)



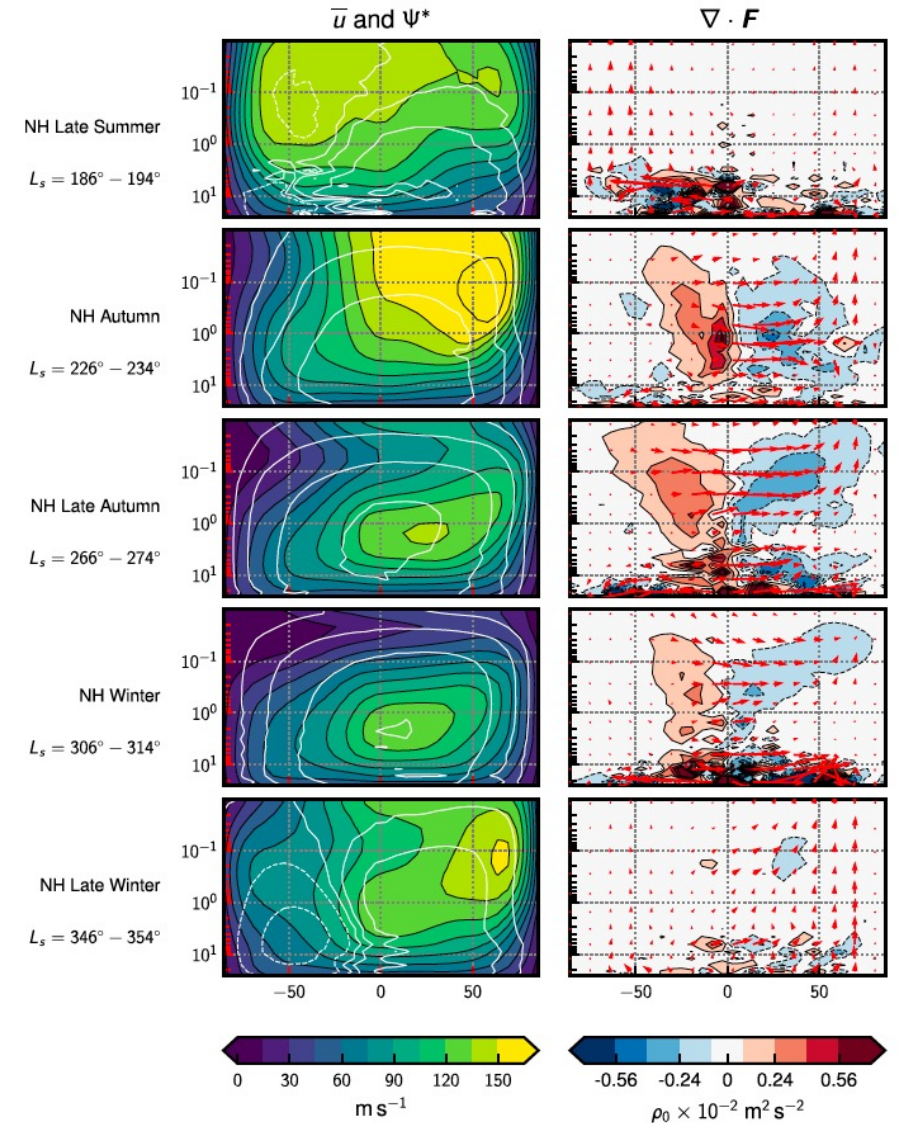
Seasonal variation

Vertical shear

Strongly stratified ($N > 0$), plus N depends strongly on altitude / pressure:



Seasonal cycle of \bar{u} , Ψ^* , and EP fluxes:



5 Basic circulation and eddy momentum fluxes

Key features:

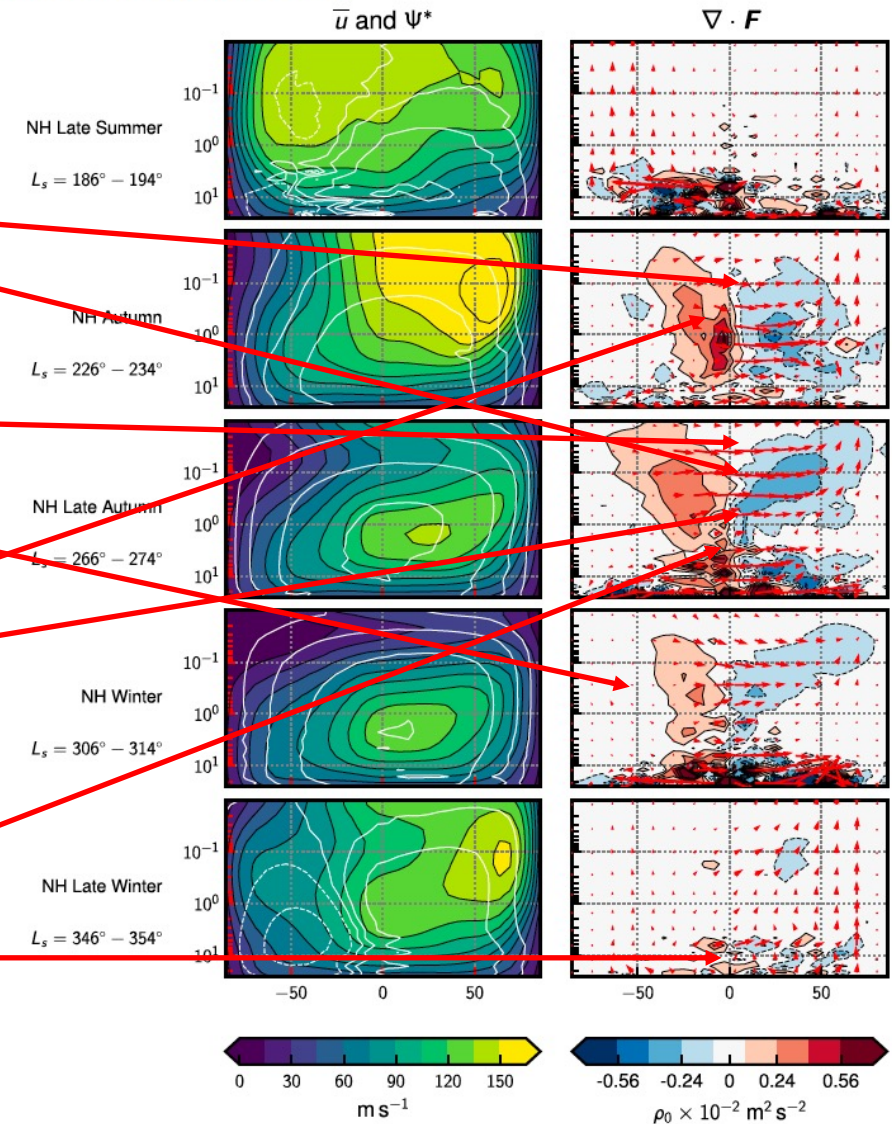
Nearly horizontal EP flux vectors where momentum transport strongest – acceleration due to horizontal component of divergence.

But! There is evidence of vertical wave propagation.
Caveat: direction unknown without knowing zonal direction of wave propagation.

Acceleration in upper stratosphere strongest with strong meridional shear.

Acceleration in lower stratosphere more consistent all year round.

Seasonal cycle of u , Ψ (TEM), and EP fluxes:



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6 Spectral analysis of equatorial waves

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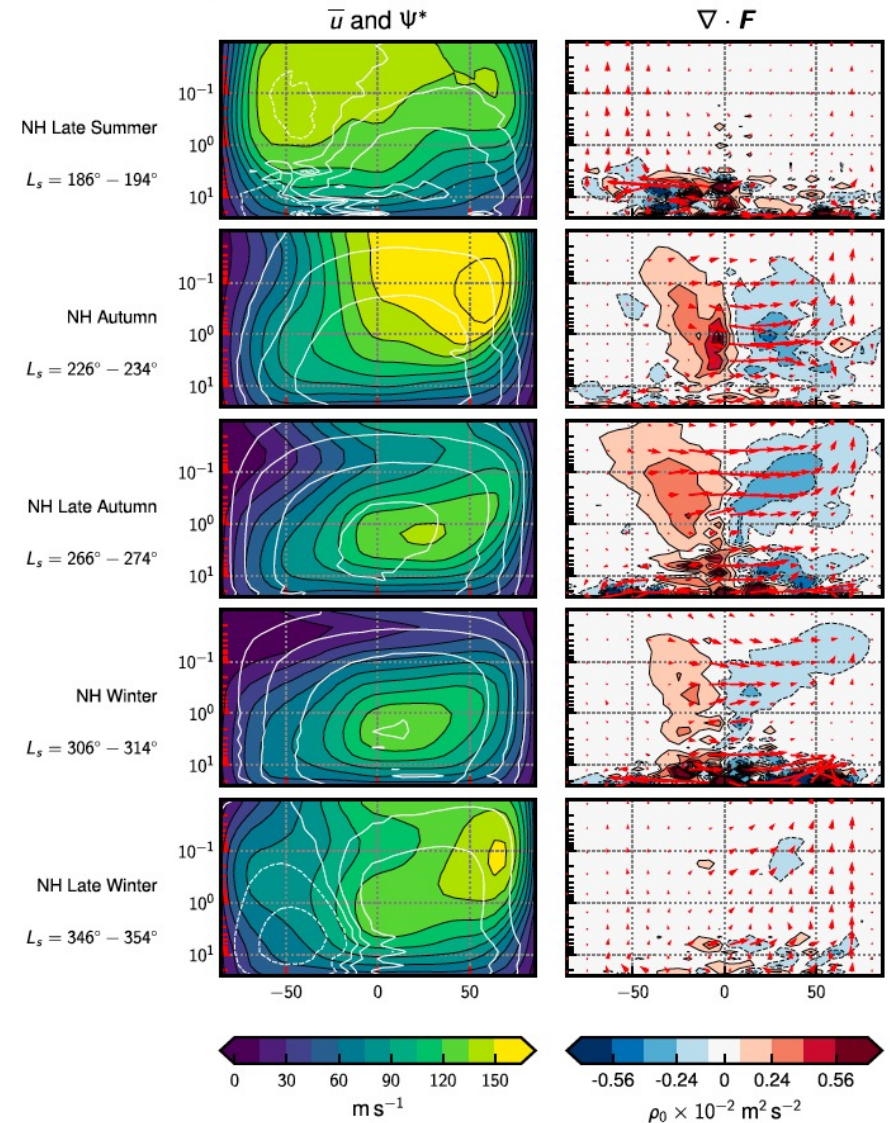
6.1 Waves in NH late autumn / early winter (Lsol 226—234)

- Upper stratosphere
- Lower stratosphere
- Vertical structure

6.2 Waves in NH winter (Lsol 266—274)

6.3 Waves in NH spring (Lsol 346—354)

6.4 Possible generation mechanisms



6 Spectral analysis of equatorial waves

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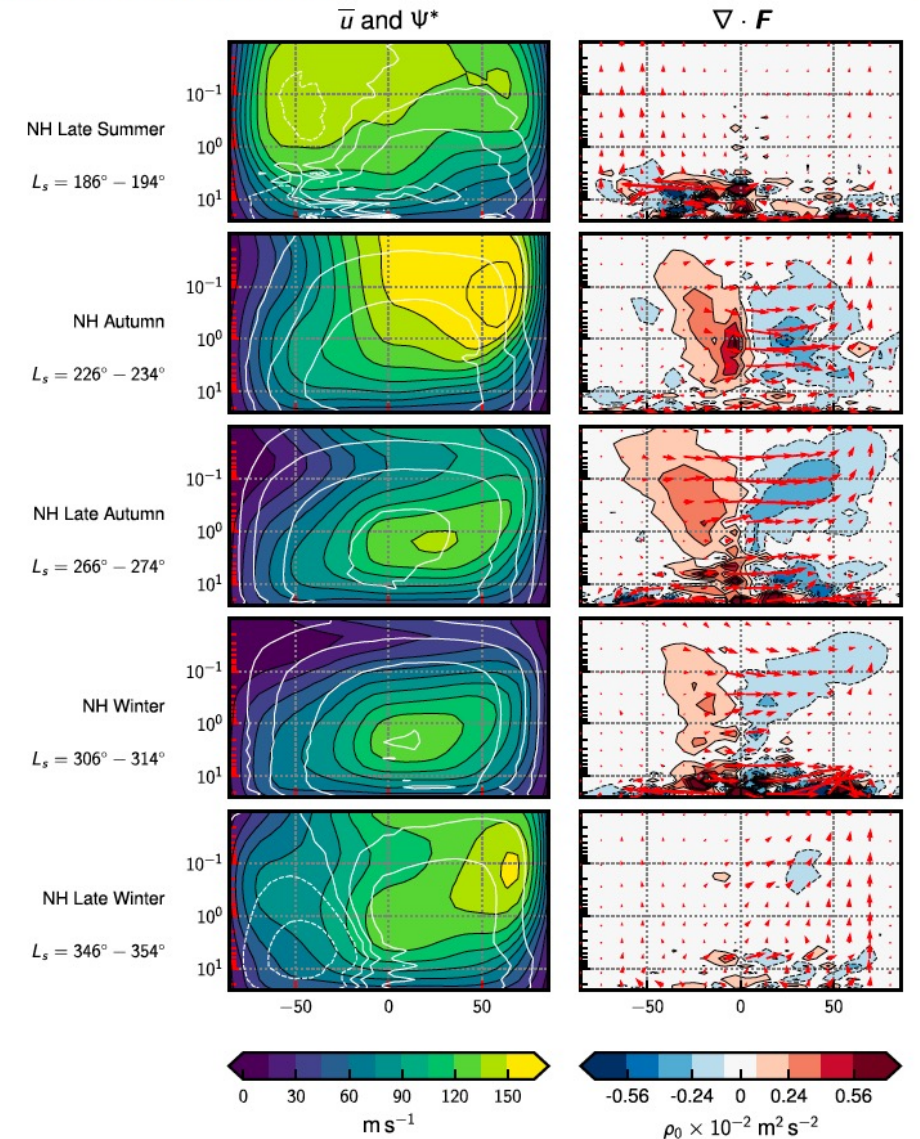
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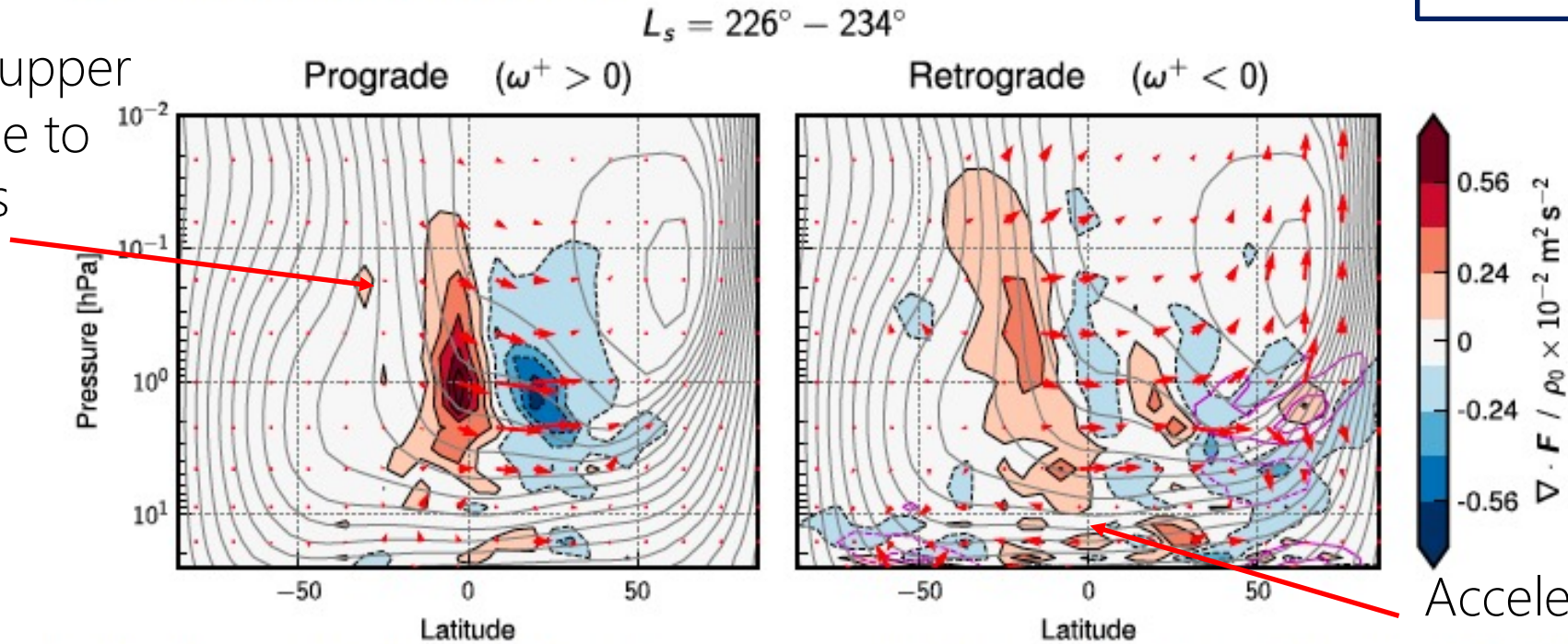


6 Spectral analysis of equatorial waves – late autumn

EP flux filtered into contributions from prograde and retrograde disturbances

$$\omega^+ = \omega - k\bar{u}$$

Acceleration in upper stratosphere due to prograde waves



Acceleration in lower stratosphere due to retrograde waves

Prograde: EP flux anti-parallel to direction of wave propagation

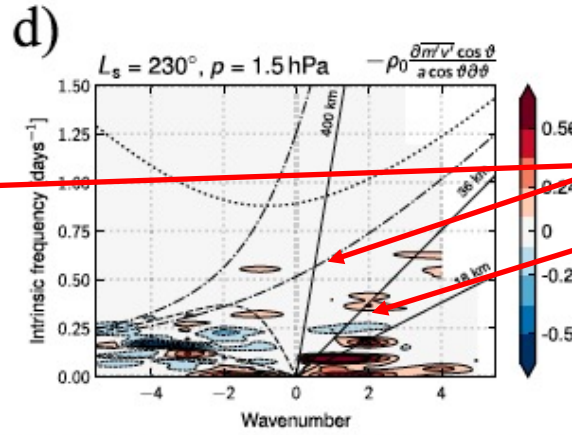
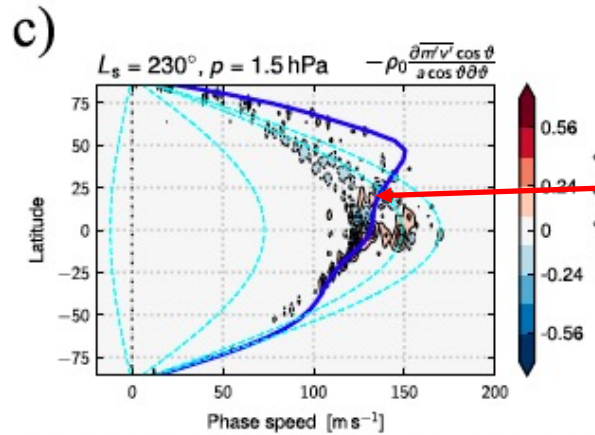
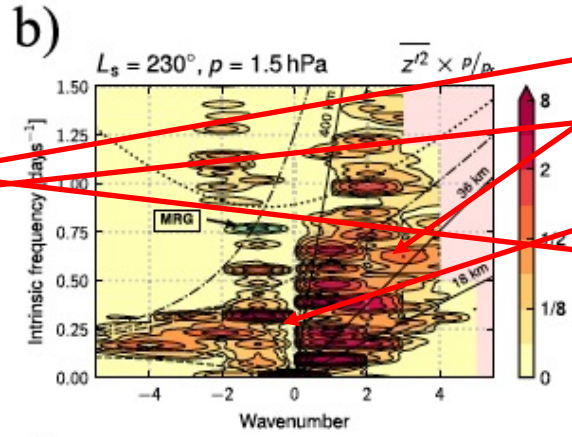
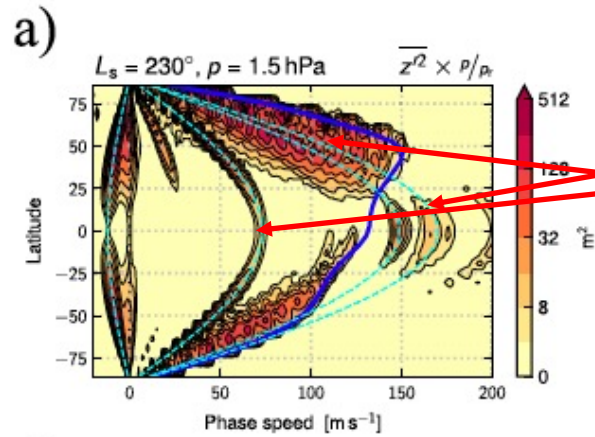
Retrograde: EP flux parallel to direction of wave propagation

Both: waves propagate upwards

(Andrews et al. 1987, Imamura 2006)

6 ... analysis of equatorial waves – late autumn, upper stratosphere

Spectral decomposition of eddy momentum flux and geopotential height



Prograde and retrograde waves present at equator

Retrograde waves in mid-latitudes

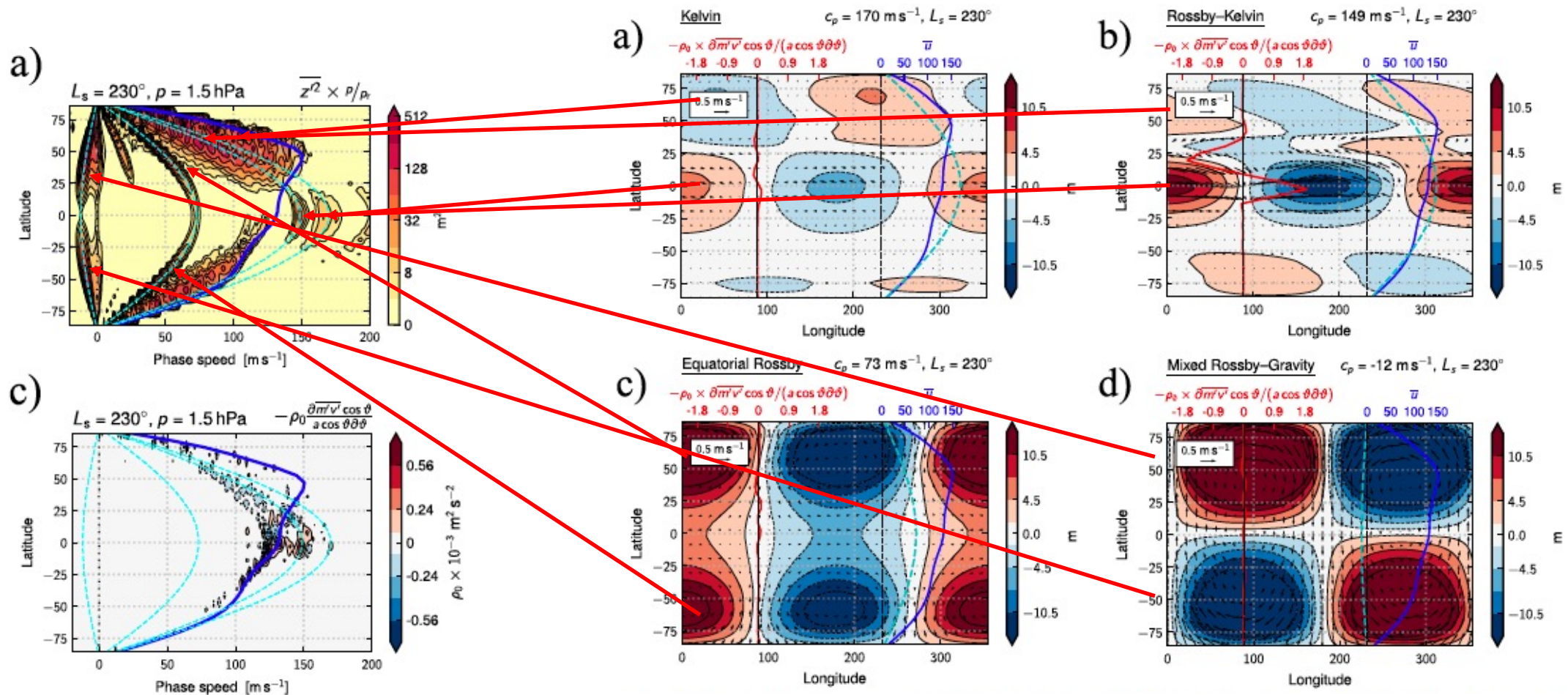
Acceleration at equator due to modes with low intrinsic frequency

- mostly prograde

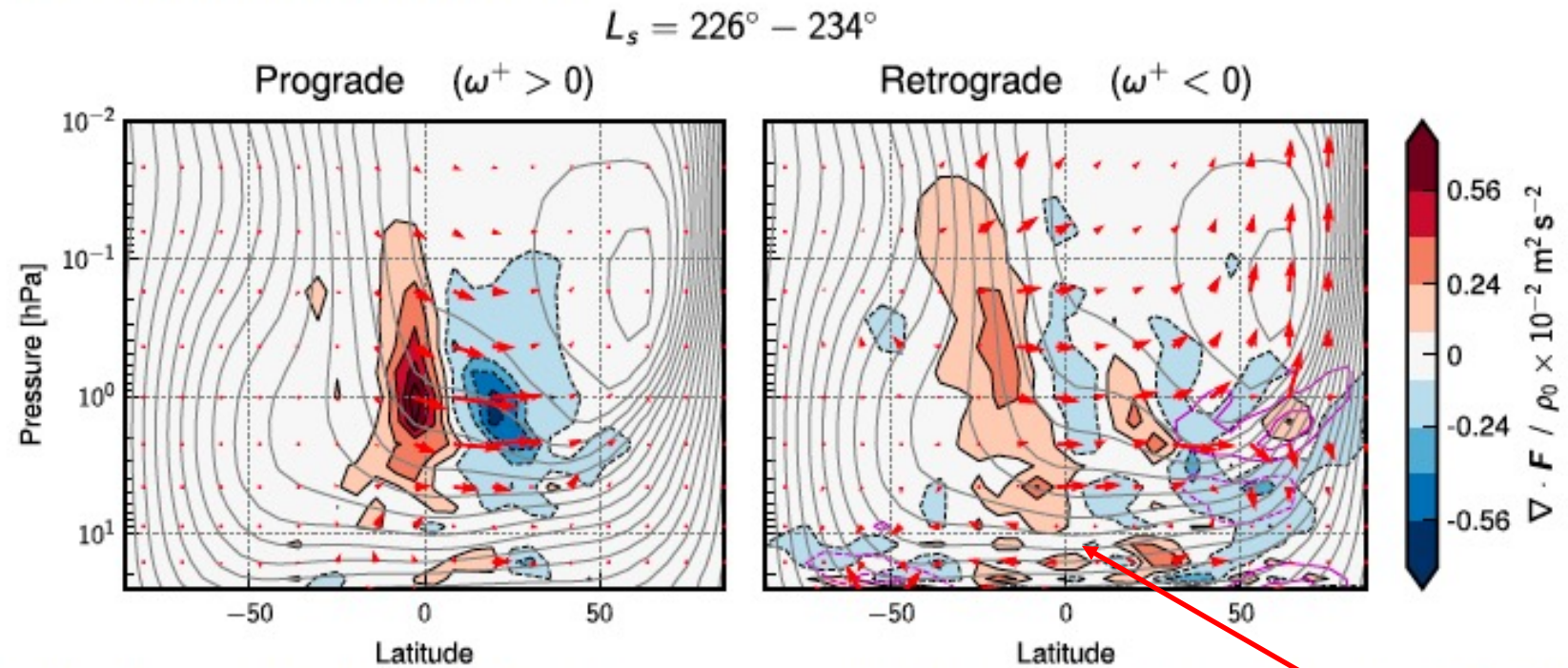
What do they look like?

6 ... analysis of equatorial waves – late autumn, upper stratosphere

Horizontal structure for selected waves



6. ... analysis of equatorial waves – late autumn, lower stratosphere

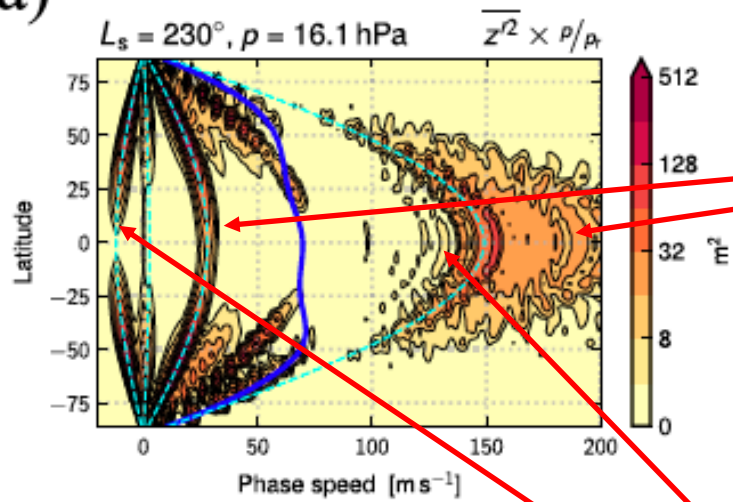


Acceleration in lower stratosphere due to retrograde waves

6 ... analysis of equatorial waves – late autumn, lower stratosphere

Spectral decomposition of eddy momentum flux and geopotential height

a)

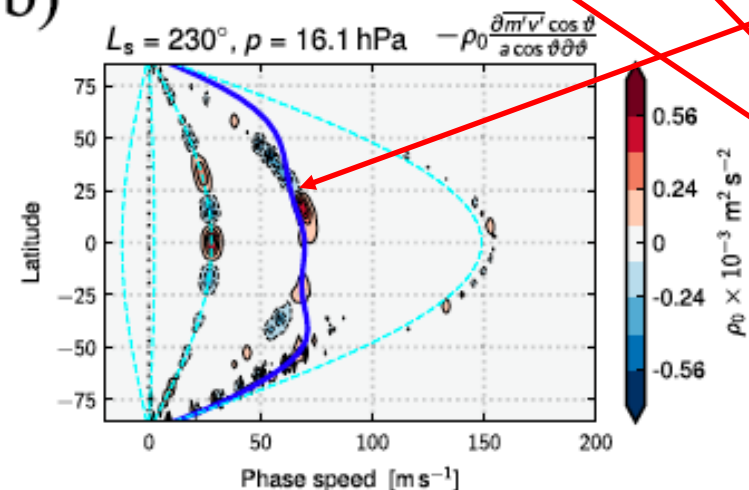


Prograde and retrograde waves present at equator, again!

Acceleration at equator due to modes with low intrinsic frequency, again!

- BUT, mostly retrograde

b)

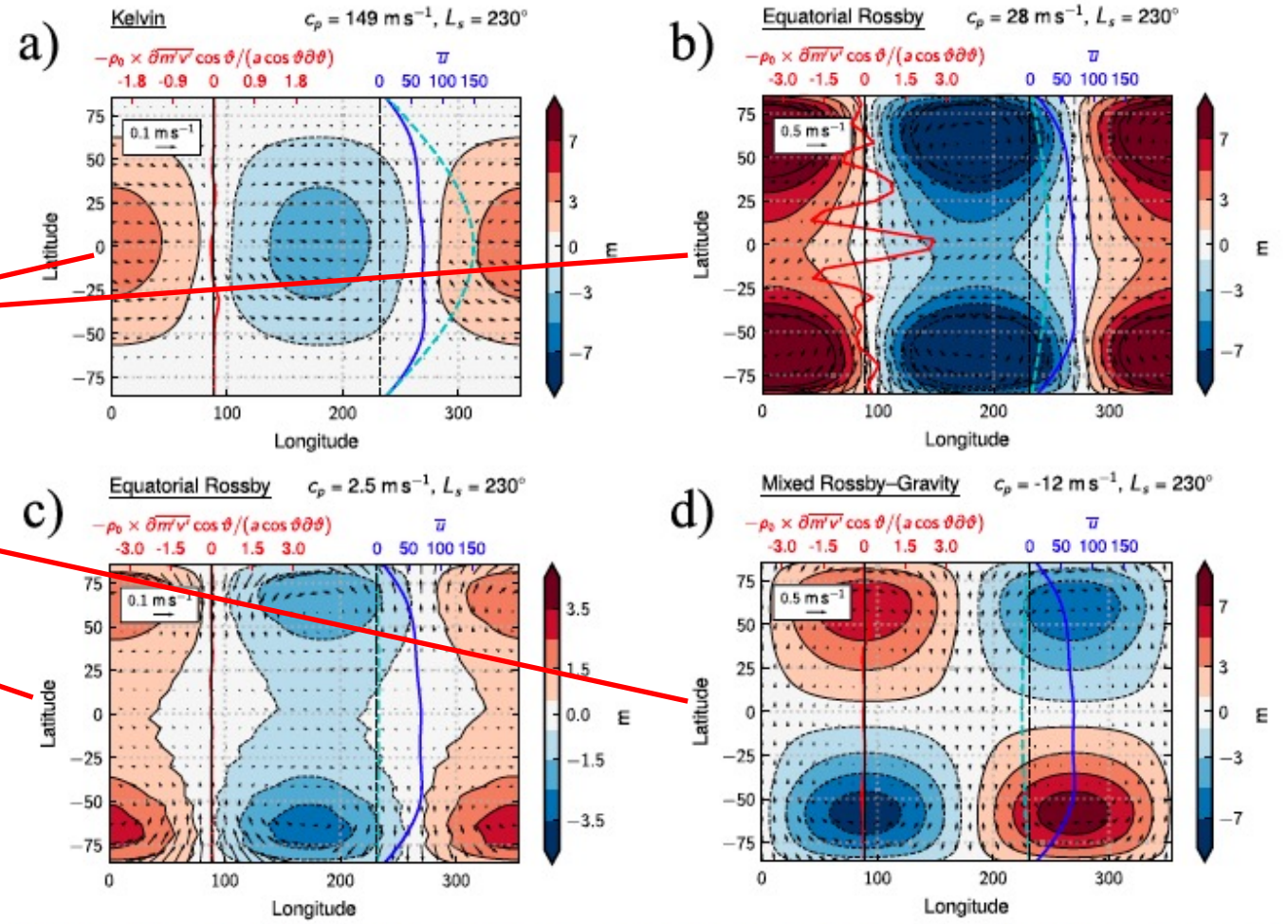
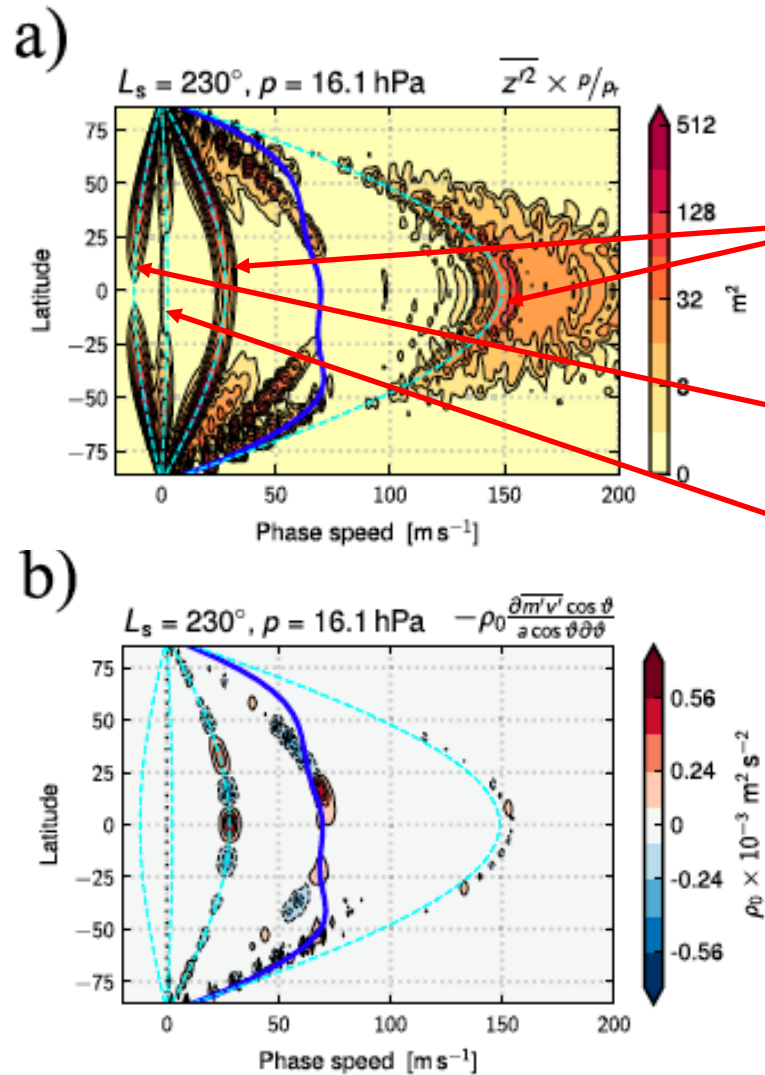


Note also signature of eastward waves, and westward wave with $c_p = -12 \text{ m s}^{-1}$, similar to upper stratosphere

What do they look like?

6 ... analysis of equatorial waves – late autumn, lower stratosphere

Horizontal structure for selected waves



Note:

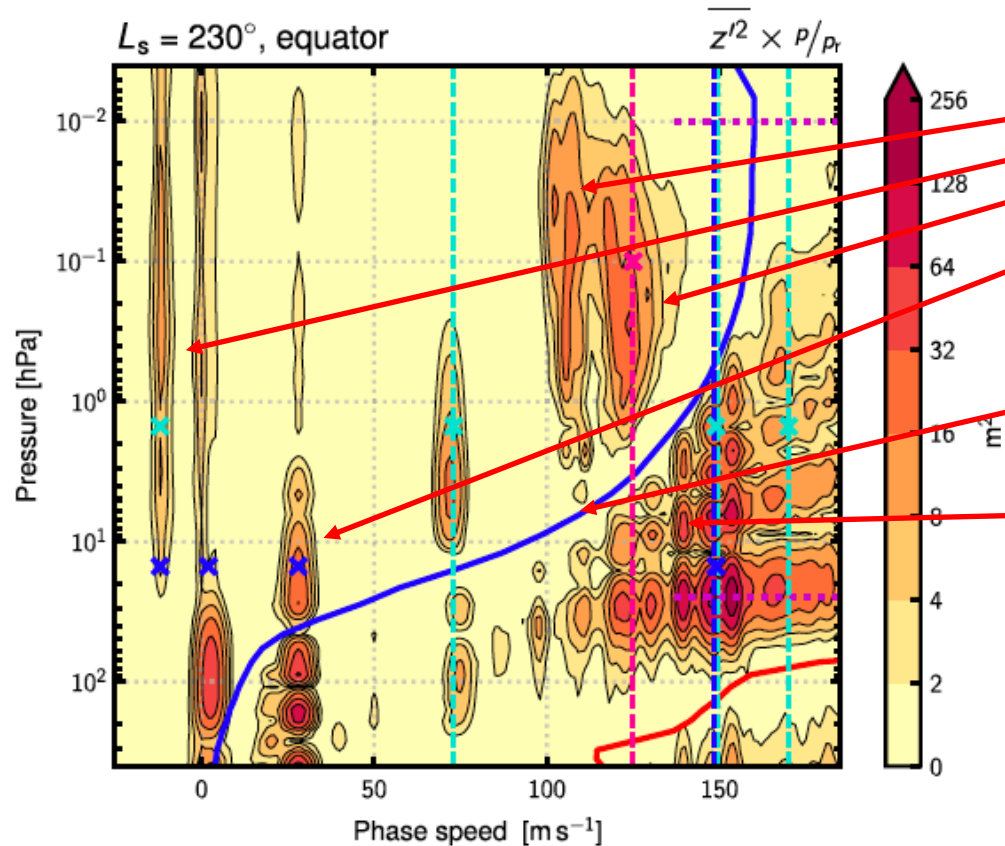
- 1) Re-appearance of MRG
- 2) Rossby—Kelvin -> pure Kelvin

6 ... analysis of equatorial waves – late autumn, lower stratosphere

What about vertical structure?

Spectral decomposition of eddy
geopotential (at equator):

A number of waves have coherent vertical
structure.



Evidence for a critical region through which
disturbances can not propagate vertically

Prograde waves in lower stratosphere
appear to be concentrated onto certain
pressure levels.

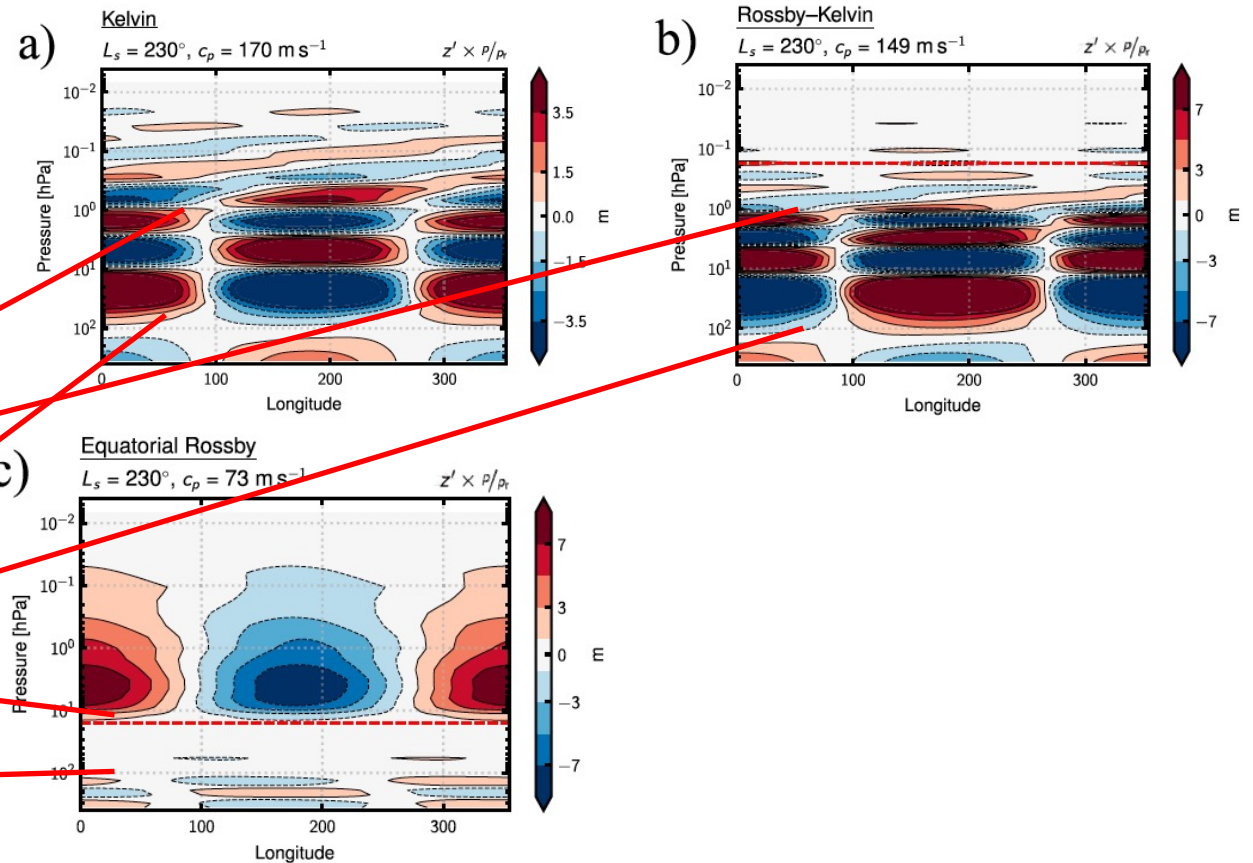
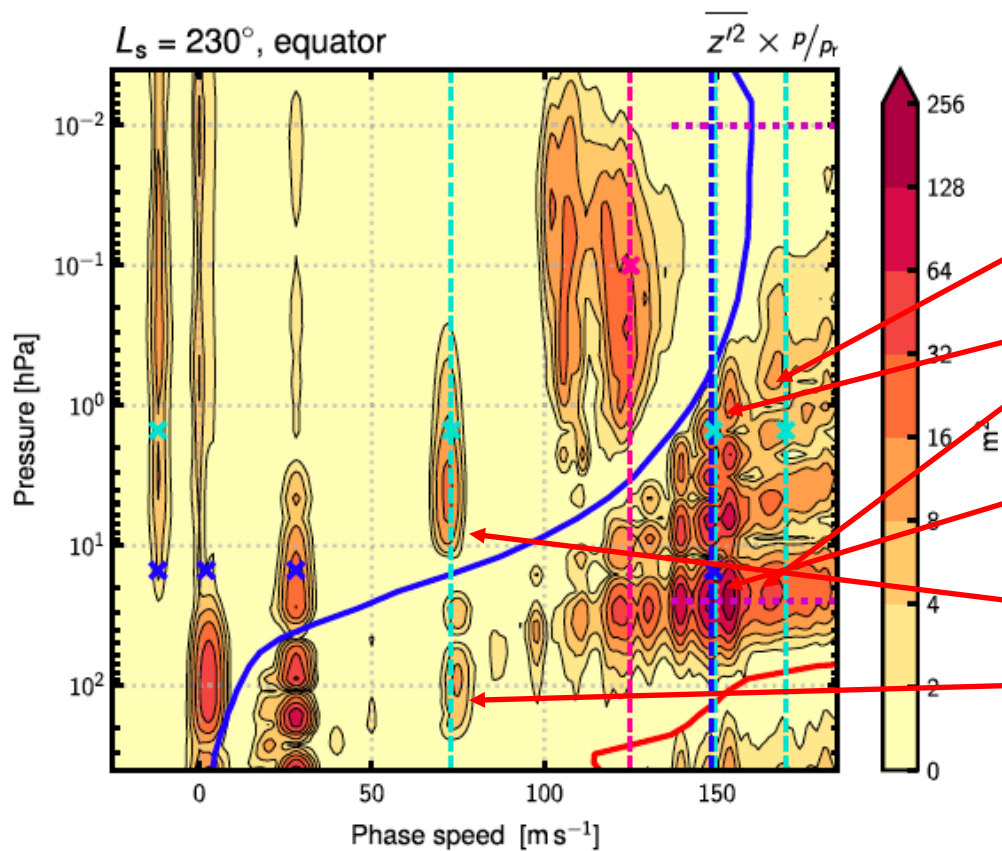
Roughly coincident with sharp gradient in
background wind and also buoyancy freq.

6 ... analysis of equatorial waves – late autumn, lower stratosphere

What about vertical structure?

What do they look like?

Spectral decomposition of eddy geopotential (at equator):

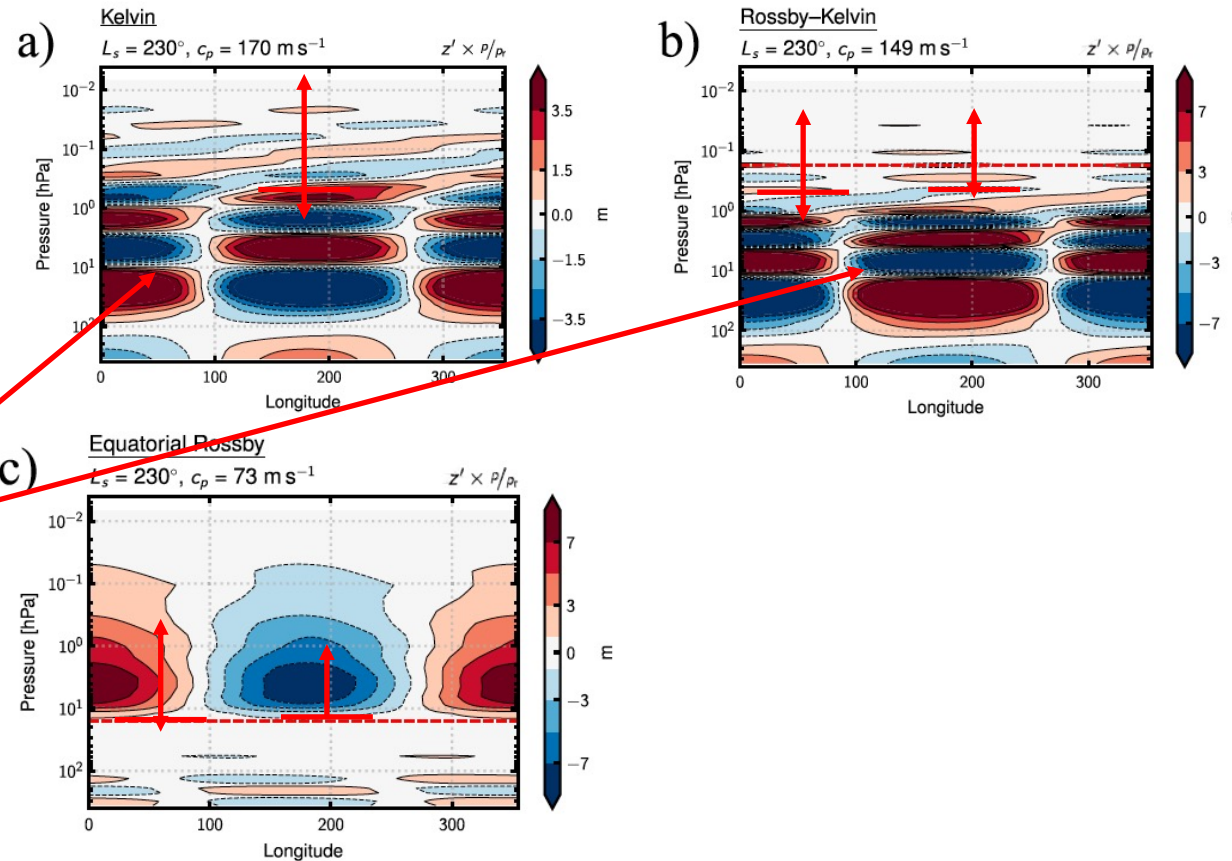


6 ... analysis of equatorial waves – late autumn, lower stratosphere

What do they look like?

Vertical propagation away from a source region

Unusual 'vertically trapped' structure for some prograde waves in lower stratosphere



6 Spectral analysis of equatorial waves

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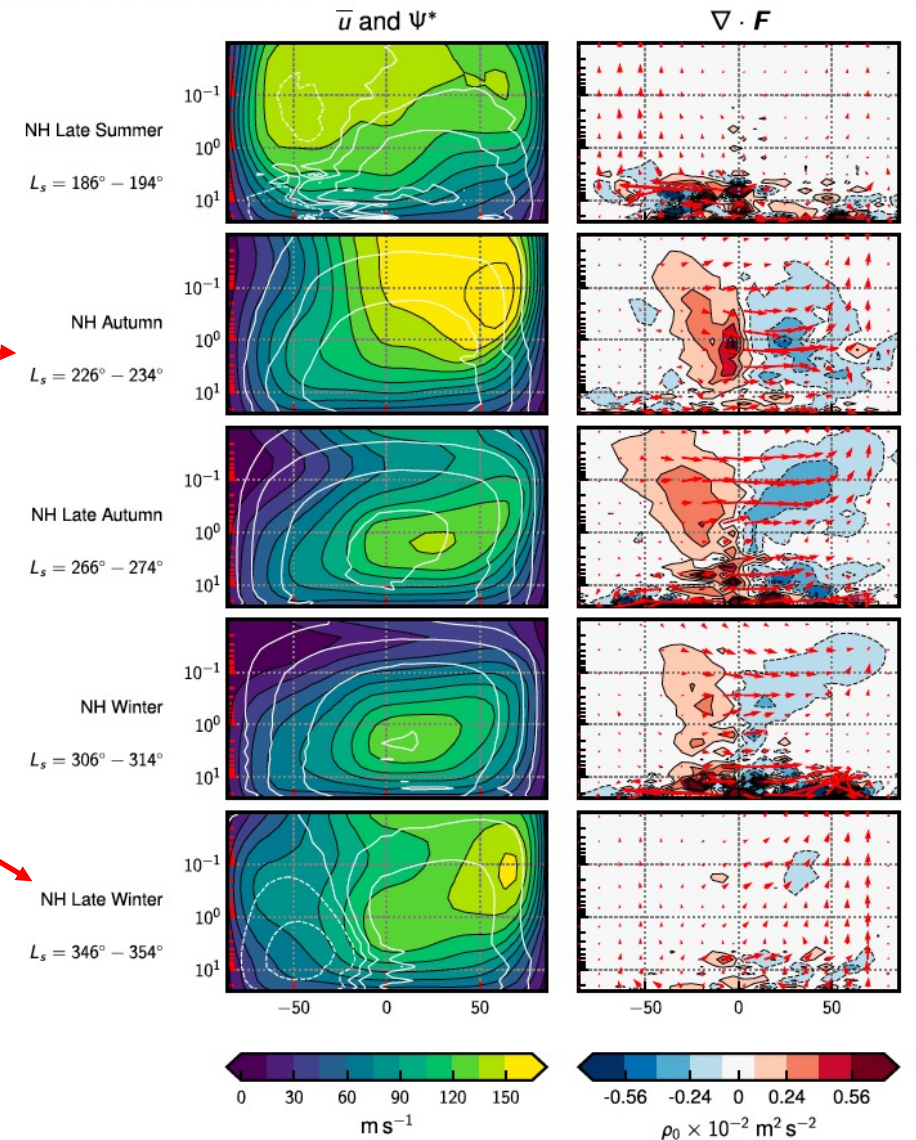
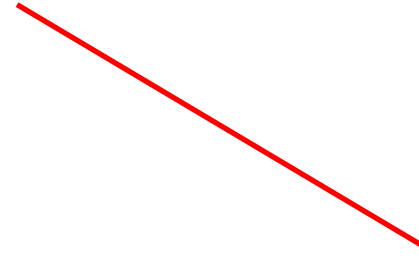
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6.4 Possible generation mechanisms



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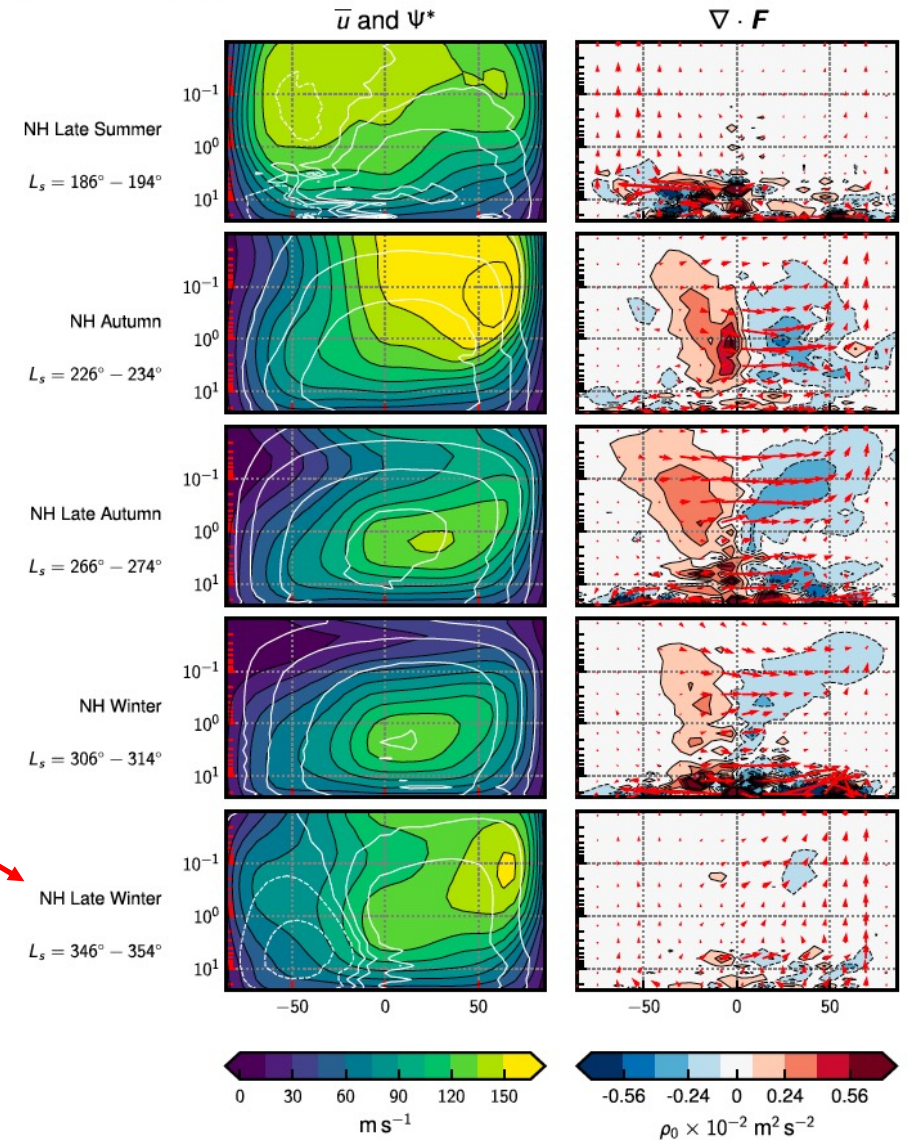
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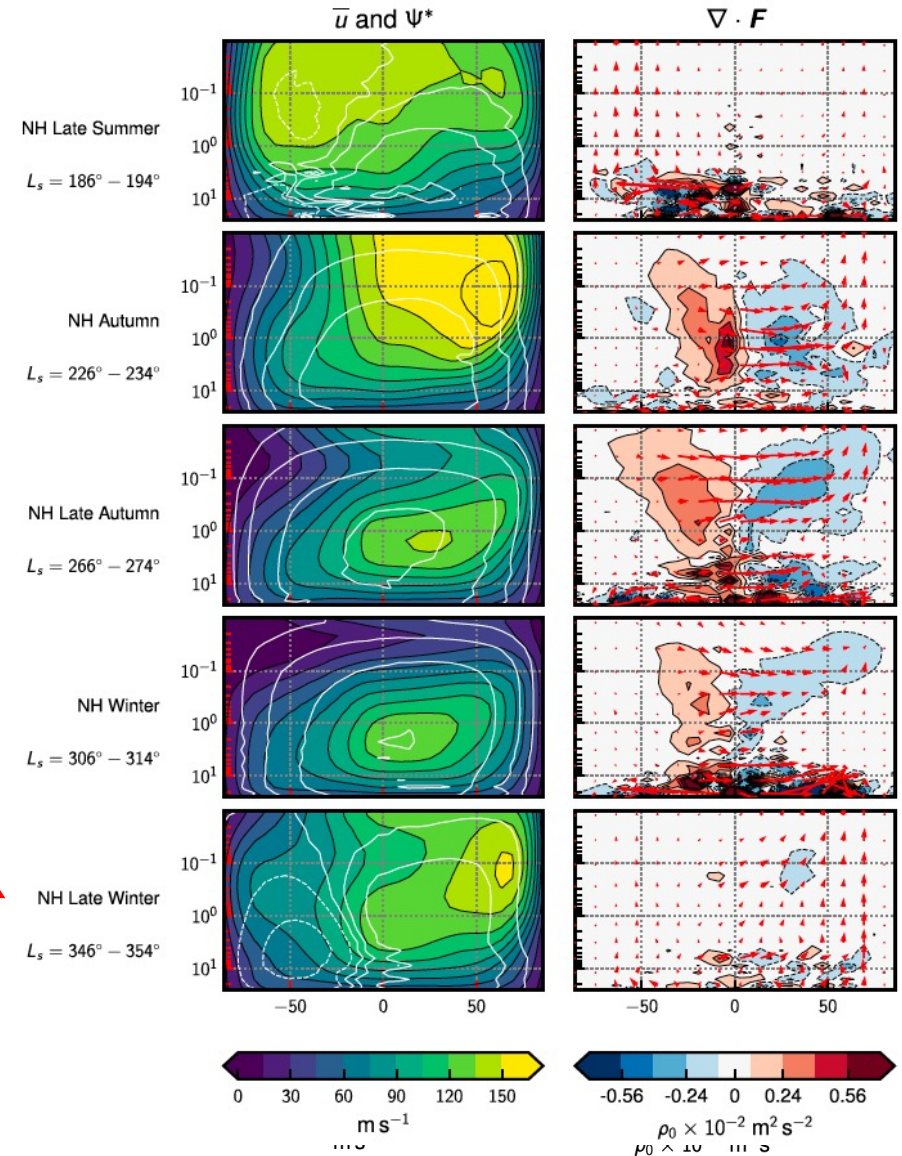
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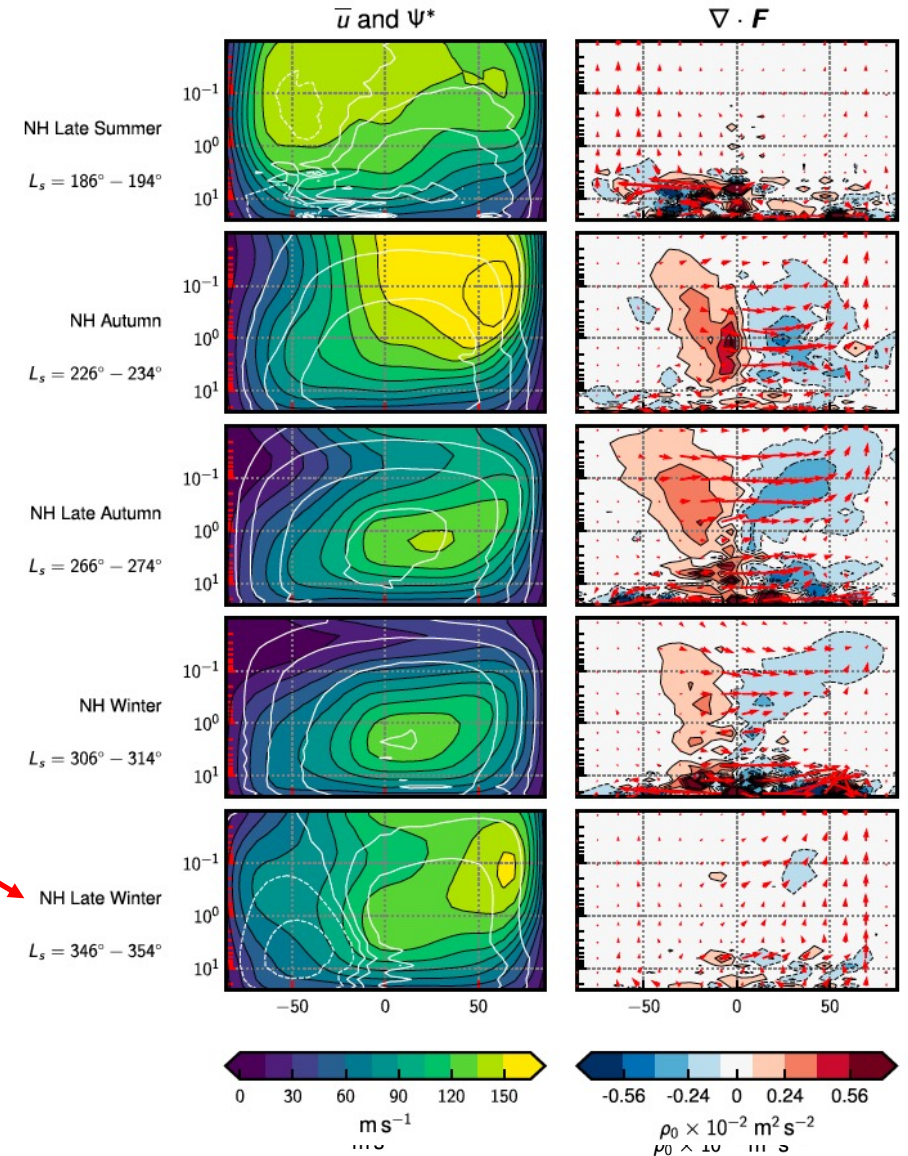
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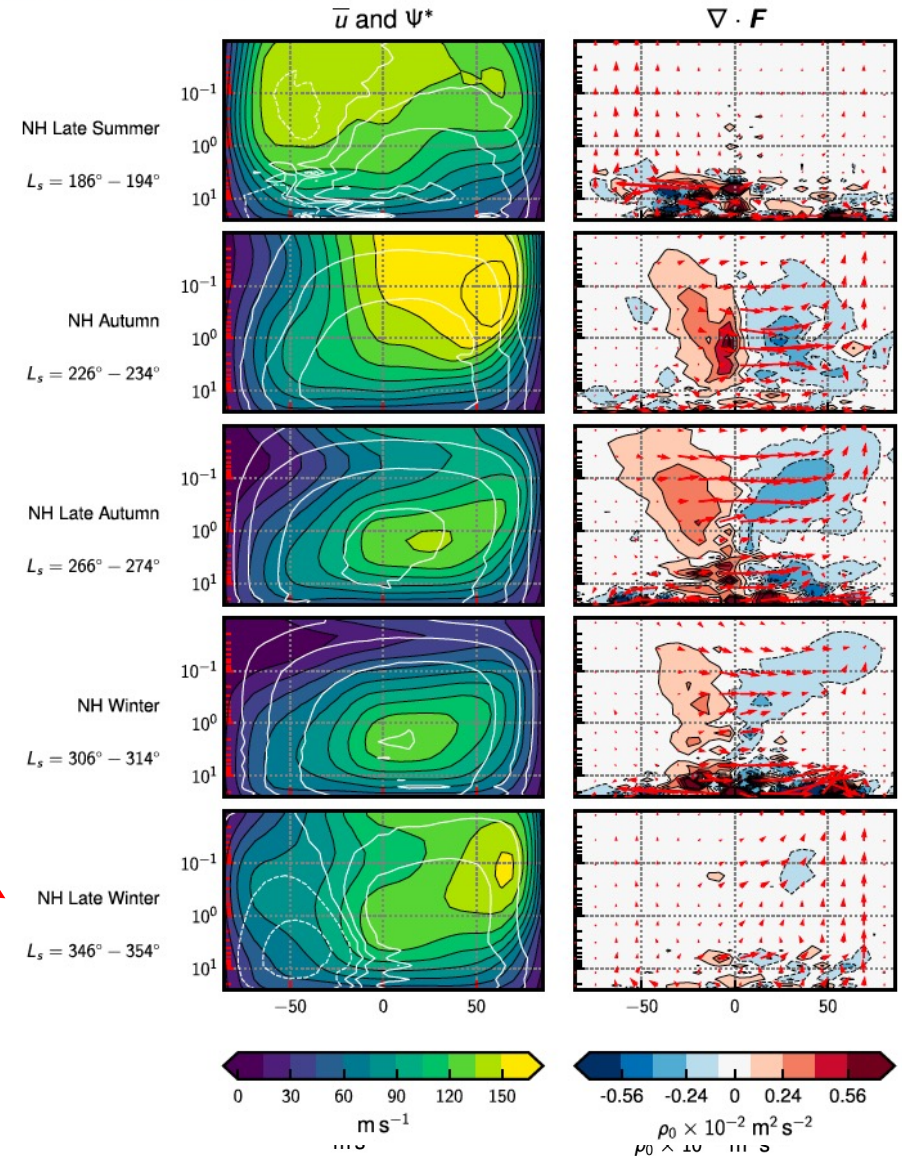
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Overview:

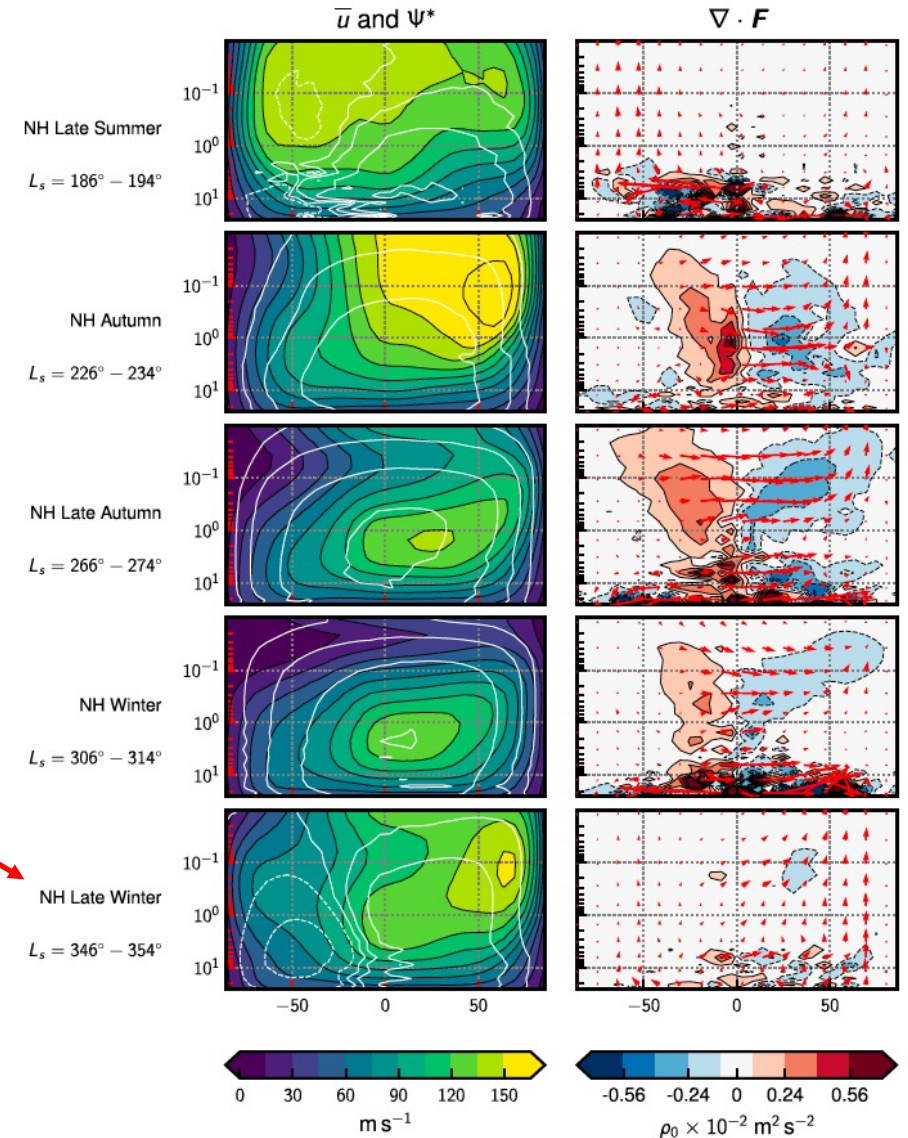
6.1 Waves in NH late autumn / early winter (L_s 226—234)

- Upper stratosphere
- Lower stratosphere
- Vertical structure

6.2 Waves in NH winter (L_s 266—274)

6.3 Waves in NH spring (L_s 346—354)

6.4 Possible generation mechanisms

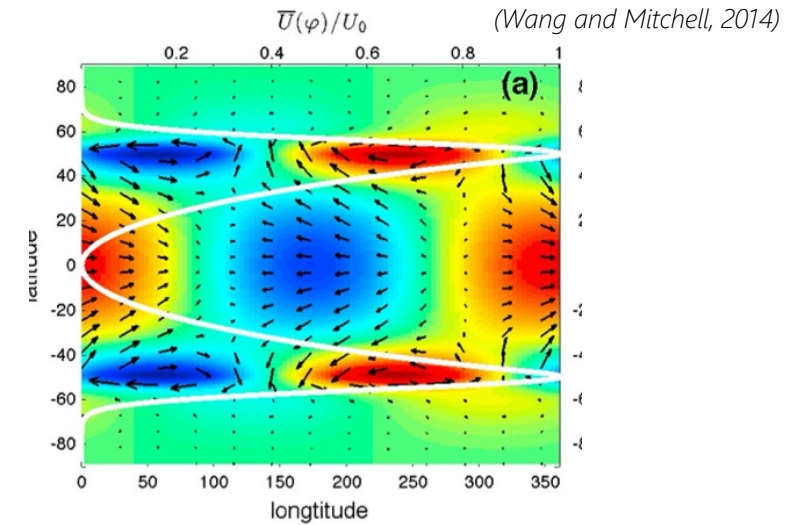


5 ... analysis of equatorial waves – generation mechanisms

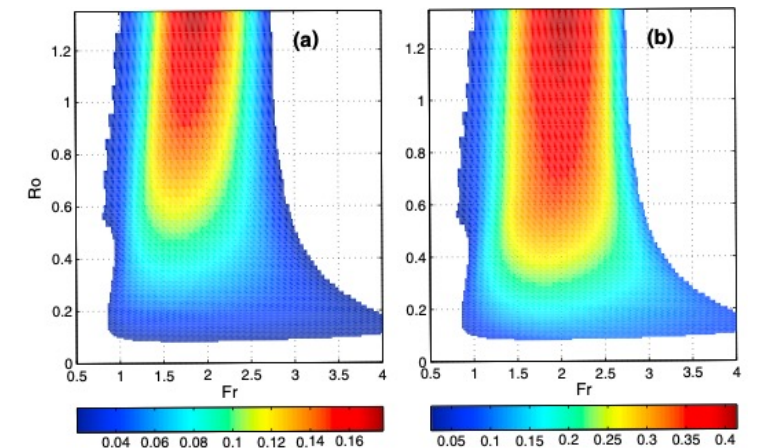
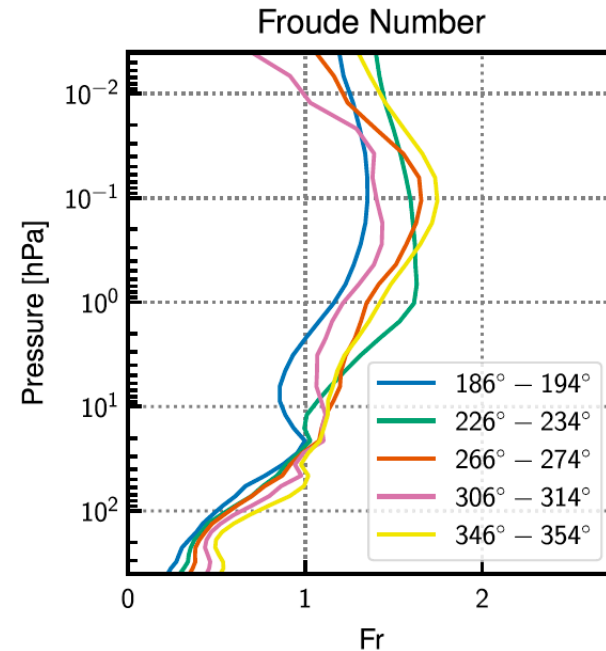
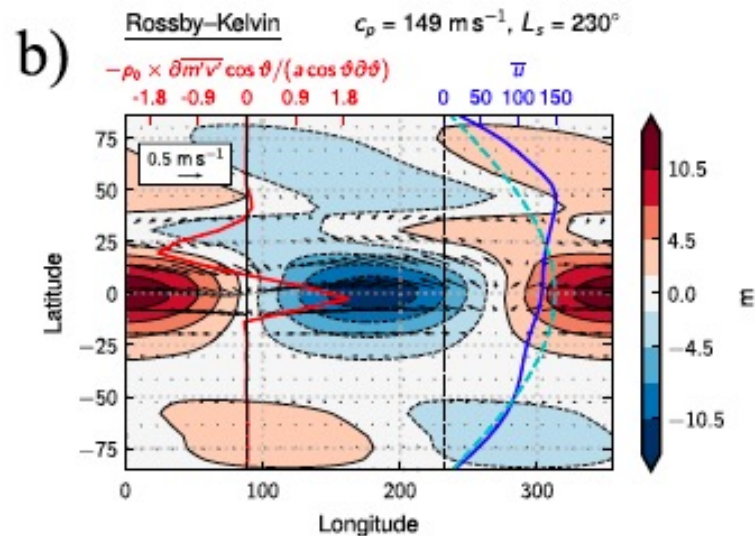
Ageostrophic, shear instability?

Occurs when
$$\text{Fr} \equiv \frac{\omega_{\text{Rossby}}}{\omega_{\text{Kelvin}}} \approx \frac{\bar{u}_{\text{ml}} / \cos \vartheta_{\text{ml}}}{\bar{u}_{\text{eq}} + \omega_{\text{Kelvin}}^+ / k} \approx 1 - 2$$

$$\omega_{\text{Kelvin}}^+ = -\frac{Nk}{[m^2 + 1/(4H^2)]^{1/2}}$$



Our work:

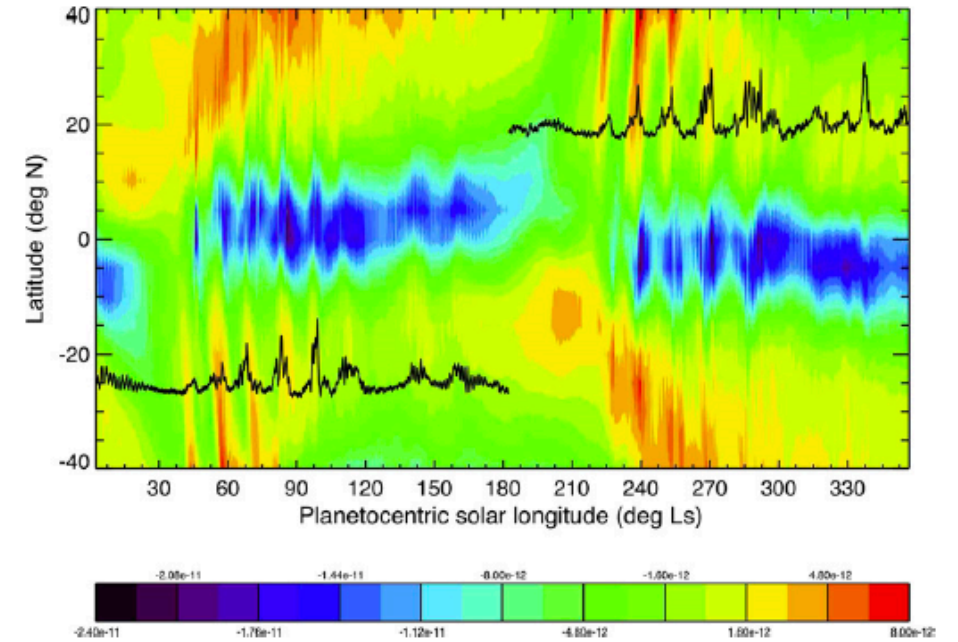
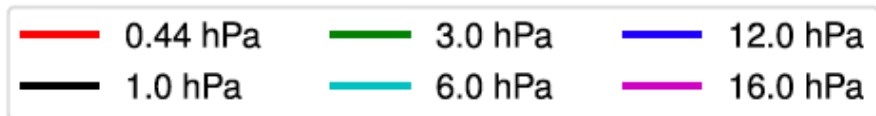
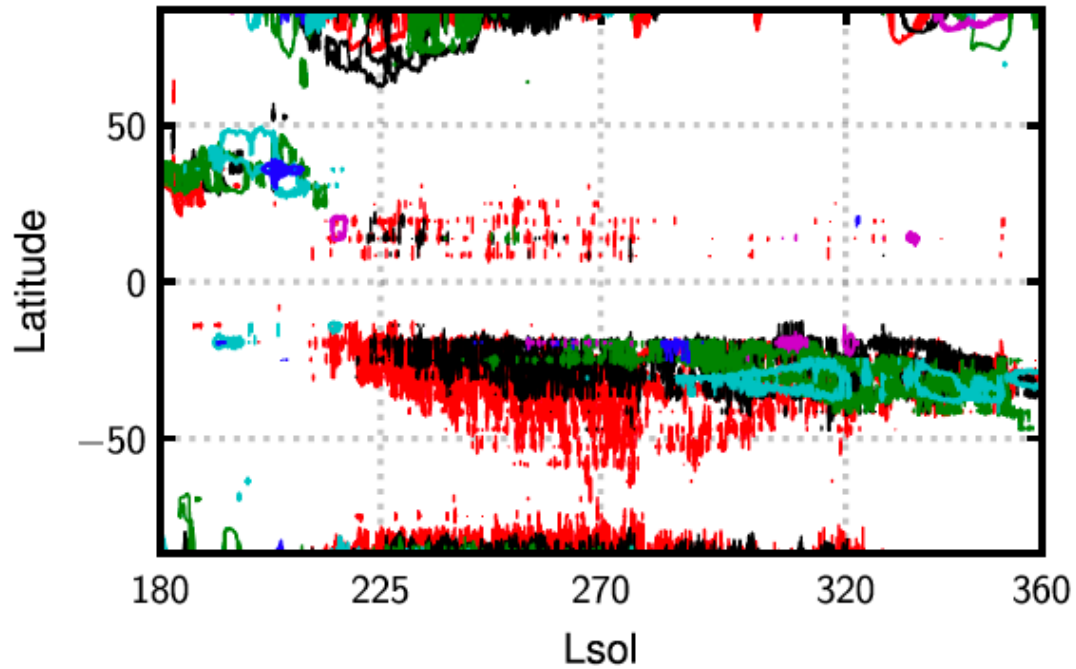


5 ... analysis of equatorial waves – generation mechanisms

What about for retrograde waves, that exist all year round?

Barotropic instability? (cf. Lebonnois et al., Newman et al.)

$$\partial \bar{Q} / \partial \vartheta = 0$$



Criterion met all year round

Contents

- 1 Key features of Titan
- 2 Equatorial superrotation
- 3 Open questions + aim
- 4 Model configuration
- 5 Basic circulation and eddy momentum fluxes
- 6 Spectral analysis of equatorial waves
- 7 Summary / discussion

Contents

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7 Summary / discussion

Which (equatorial) wave modes are supported in Titan's stratosphere, and what is their spatial structure? - many: K, RK, ER, MRG (IG waves not shown)

Which contribute to maintaining equatorial superrotation?

- mostly RK in upper strat, ER in lower strat

What generates them?

- consistent with ageostrophic shear instability, and barotropic instability

What relevance do the results from idealised modelling have?

- some!

Despite differences, Titan resembles idealized model spin-up in upper stratosphere, and equilibrated idealized model in lower stratosphere....

Qs: Are waves properly resolved in vertical?

Is the vertically trapped structure real?

What wave modes do we expect (e.g., from instability analysis) from an asymmetric background with strong vertical shear?

Should there be more vertical momentum transport? (cf. the wind drop-off region on real Titan)

The End

An M-G-M
Tom and Jerry
CARTOON

MADE IN HOLLYWOOD, U.S.A.





The End

An M-G-M

Saturn and Titan

CARTOON

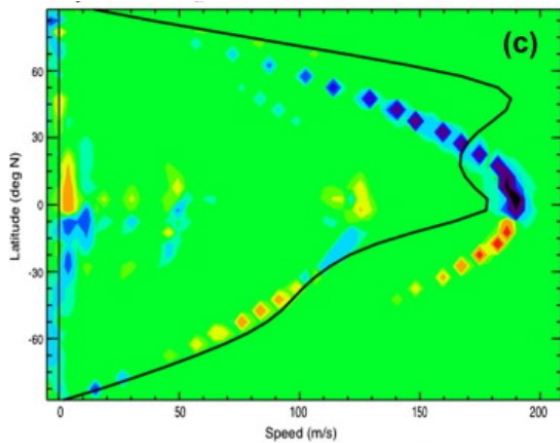
MADE IN HOLLYWOOD, U.S.A.

Equatorial superrotation – what about Titan?

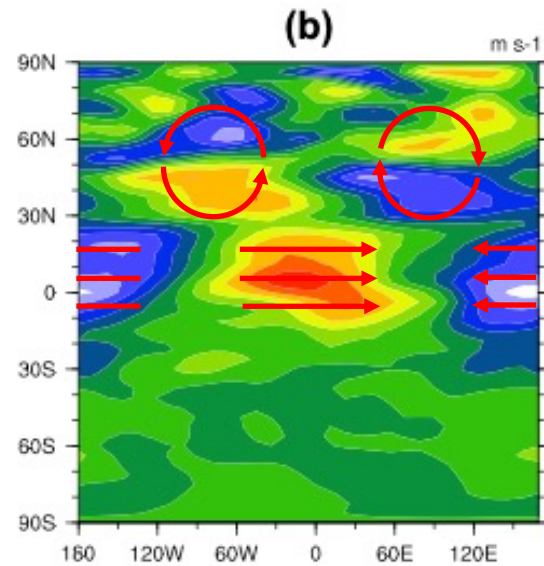
Situation on Titan complicated by seasonal cycle and latitudinally asymmetric shear

This leads to a seasonal cycle in momentum budget...

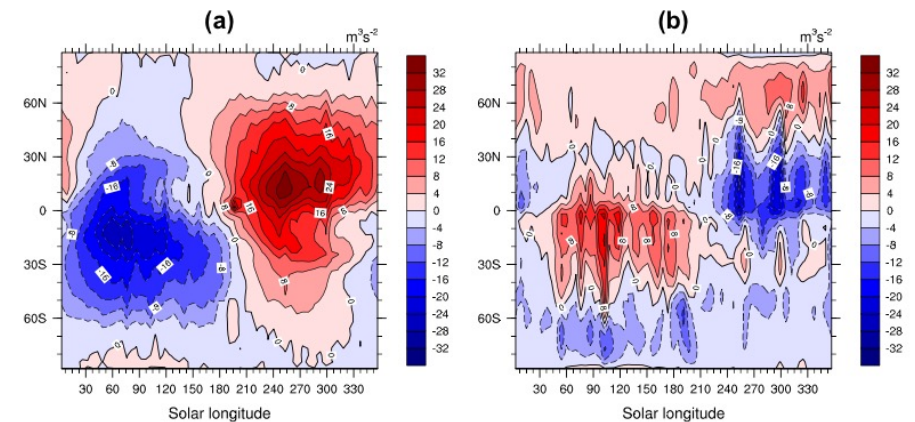
What are the eddies like?



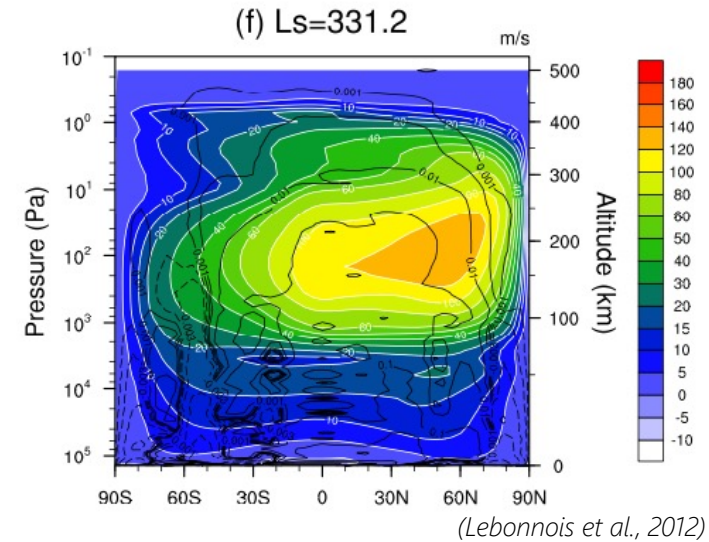
(Newman et al., 2011)



(Lebonnois et al., 2012)



(Lebonnois et al., 2012)



(Lebonnois et al., 2012)

These authors suggest regular barotropic instability as origin for the waves