Gravity currents as a test case: do simple mathematical models produce good physical insights, or do good insights generate powerful models?

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Abstract

A gravity current appears when fluid of one density, ρ_c, propagates into another fluid of a different density, ρ_a, and the motion is mainly in the horizontal direction. A gravity current is formed when we open the door of a heated house and cold air from outside flows over the floor into the less dense warm air inside. A gravity current is formed when we pour honey on a pancake and we let it spread out on its own. Gravity currents originate in many natural and industrial circumstances and are present in the atmosphere, lakes and oceans as winds, cold or warm streams or currents, polluted discharges, etc. A gravity current which propagates inside a stratified fluid (about the neutral buoyancy lever rather than along a boundary) is called "intrusion"; an important application is the spreading in the atmosphere of umbrella clouds generated by volcanic eruptions (see [1]).

Simple qualitative consideration and observations indicate that the gravity current is a very complex, multi-faced, and parameter-rich physical manifestation. Nevertheless, the gravity current also turns out to be a modeling-friendly phenomenon. Indeed, visualizations of the real flow field reveal an extremely complicated three-dimensional motion, with an irregular interface, billows, mixing, and instabilities. On the other hand, there are mathematical models based on a long line of assumptions such as hydrostatic pressure, sharp interface, Boussinesq system, thin layer, idealized release conditions (see [2]). This enables us to determine the behavior of the averaged variables entirely from theoretical considerations and solutions of "mathematical models" which are significantly simpler than the full set of governing equations (the Navier-Stokes system).

The lecture gives a brief presentation of some typical models and solutions. We see that: (a) Qualitatively, the theory is able to provide the governing dimensionless parameters and the salient features of the various flow regimes; and (b) Quantitatively, the simple models predict velocities of propagation which agree with experiments within a few percent, sometimes within the range of the experimental errors. The fact that such simple models give useful results merits attention: is the predictive power of the model a consequence of postulated mathematical simplicity, or is it rather a result of well-selected physical components? Some tentative answers to this universally relevant question, as provided by the test-case of gravity currents and intrusions, are discussed.

References

[1] Y. J. Suzuki and T. Koyaguchi. A three-dimensional numerical simulation of spreading umbrella clouds. *J. Geophys. Res.*, 114:1–18, 2009.

[2] M. Ungarish. *An Introduction to Gravity Currents and Intrusions.* Chapman & Hall/CRC press, Boca Raton London New York, 2009.