6. Radiative Transfer in Optically Thick Atmospheres

Gaussian Quadrature ガウス求積法

- **Optimum discrete quadrature for spherical harmonic functions**
- **Assume only two streams in *m*-axis. Asymmetry factor**
- **2 term approximation**

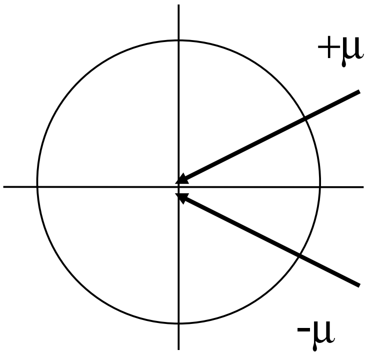
$$I = \int_{-1}^{1} f(\mu) d\mu \approx \sum_{i=1}^{N} f(\mu_i) w_i$$

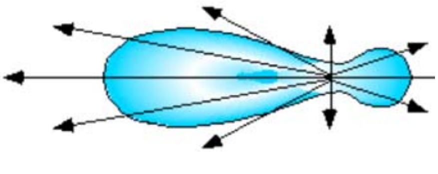
$$w = w_1 = w_2 = 1, \quad \mu = -\mu_1 = \mu_2 = \frac{1}{\gamma}$$

$$\gamma = \sqrt{3}, \quad \gamma = 1.66, \quad 1.73, \quad 2$$

$$f = 1, x, x^2, x^3$$

$$P(\cos\Theta) \approx \frac{1}{4\pi}(1 + 3g\cos\Theta)$$





Flux integration in the two stream approximation

$$\pm\mu\frac{dL(\tau,\pm\mu,\phi)}{d\tau}=-L(\tau,\pm\mu,\phi)$$
$$+\,\omega\int_{-1}^{1}d\mu'\int_{0}^{2\pi}d\phi'\frac{1}{4\pi}\{1+3g[\pm\mu\mu'+\sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\phi-\phi')]\}L(\tau,\mu',\phi')+(1-\omega)B(T)$$

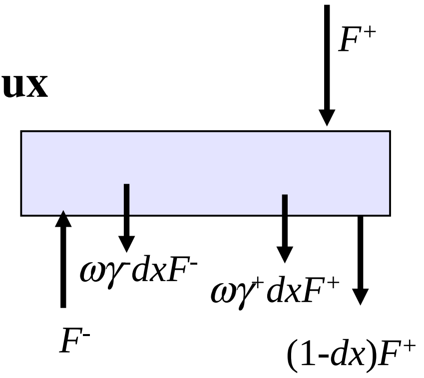
$$\int_0^1 d\mu\mu\int_0^{2\pi}d\phi f(\mu,\phi)\rightarrow 2\pi\bar{f}(\mu),\quad \int_0^1 d\mu\mu\int_0^{2\pi}d\phi L(\pm\mu,\phi)\approx 2\pi\mu\bar{L}(\pm\mu)=F^{\pm}$$

$$\int_0^1 d\mu\mu\int_0^{2\pi}d\phi\int_{-1}^{1}d\mu'\int_0^{2\pi}d\phi'\frac{1}{4\pi}\{1+3g[\pm\mu\mu'+\sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\phi-\phi')]\}L(\tau,\mu',\phi')$$
$$=\int_0^1 d\mu\mu\int_{-1}^{1}d\mu'\frac{1}{2}(1\pm 3g\mu\mu')2\pi\bar{L}(\tau,\mu')=\frac{1}{2}(1\pm 3g\mu^2)F^{+}(\tau)+\frac{1}{2}(1\mp 3g\mu^2)F^{-}(\tau)$$

$$\int_0^1 d\mu\mu\int_0^{2\pi}d\phi B(T)=\pi B(T)$$

Flux transfer equation in the two stream approximation

- **Up and down scatter coefficients**
- **Total flux and net flux**
- **No flux convergence if ω=1: Constant net flux**



$$+\frac{dF^{+}}{dx}=-F^{+}+\omega(\gamma^{+}F^{+}+\gamma^{-}F^{-})+(1-\omega)\pi B(T)$$
$$-\frac{dF^{-}}{dx}=-F^{-}+\omega(\gamma^{-}F^{+}+\gamma^{+}F^{-})+(1-\omega)\pi B(T)$$
$$\gamma^{\pm}=\frac{1\pm 3g\mu^2}{2}=\frac{1\pm g}{2},\quad cf.\quad \gamma_{\pm}(\mu_0)=\int_0^1 d\mu\int_0^{2\pi}d\phi P(\cos\Theta_{\pm})=\frac{1}{2}(1\pm\frac{3}{2}g\mu_0)$$
$$\Psi=F^{+}(x)+F^{-}(x),\quad \Phi=F^{+}(x)-F^{-}(x)$$
$$\frac{d\Phi(x)}{dx}=(1-\omega)[-\Psi(x)+2\pi B],\quad \frac{d\Psi(x)}{dx}=-(1-\omega g)\Phi(x)$$

Non absorbing atmosphere

$$\omega = 1, \quad B = 0 \rightarrow \Phi(x) = \Phi, \quad \Psi(x) = -(1-\omega g)\Phi x + C$$

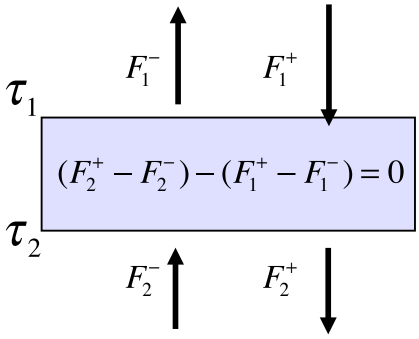
$$F^{+}(x) = \frac{1}{2}[-(1-g)\Phi x + C + \Phi], \quad F^{-}(x) = \frac{1}{2}[-(1-g)\Phi x + C - \Phi]$$

$$F^{+}(0) = F_0\mu_0, \quad F^{-}(x_b) = 0$$

$$\frac{1}{2}[C + \Phi] = F_0\mu_0, \quad \frac{1}{2}[-(1-g)\Phi x_b + C - \Phi] = 0$$

$$\Phi = \frac{F_0\mu_0}{1 + (1-g)x_b / 2}, \quad C = 2F_0\mu_0 - \Phi$$

$$r = \frac{F^{-}(0)}{F_0\mu_0} = \frac{\frac{1}{2}(C-\Phi)}{F_0\mu_0} = 1 - t, \quad t = \frac{1}{1 + \gamma^{-}\gamma\tau_b}$$



- Optical thickness of the cloud layer is obtained from transmission or reflection measurements.
- Cloud is bright to the human's eyes!

Absorbing atmosphere

- **General solution and Boundary condition**
- **Diffusion exponent and similarity parameter**

$$\frac{d^2\Psi(x)}{dx^2} = -(1-\omega g)(1-\omega)[-\Psi(x) + 2\pi B]$$

$$\Psi(x) = C_+ e^{kx} + C\_e^{-kx}$$

$$\Phi(x) = -\frac{1}{1-\omega g}\frac{d\Psi(x)}{dx} = -\frac{k}{1-\omega g}[C_+ e^{kx} - C\_e^{-kx}] = -s[C_+ e^{kx} - C\_e^{-kx}]$$

$$F^{+}(0) = \mu_0 F_0, \quad F^{-}(\tau_g) = 0$$

$$k = \sqrt{(1-\omega g)(1-\omega)}, \quad s = \sqrt{\frac{1-\omega}{1-\omega g}}, \quad E = \mathrm{e}^{-k\gamma\tau_g}, \quad \sigma = \frac{1-s}{1+s}$$

$$r = \frac{F^{-}(0)}{\mu_0 F_0} = \sigma(1 - tE), \quad t = (1-\sigma^2)E$$

$$\varepsilon = \frac{F^{+}(\tau_g)}{\pi B} = \frac{F^{-}(0)}{\pi B} = (1-\sigma)[1-(1+\sigma)E]$$

$$r + t + \varepsilon = 1$$

- Ground surface becomes very dark in near-IR region if we have clouds.
- Cloud reflectivity saturates rapidly with increasing cloud optical thickness.

Similarity parameter of cloud particles

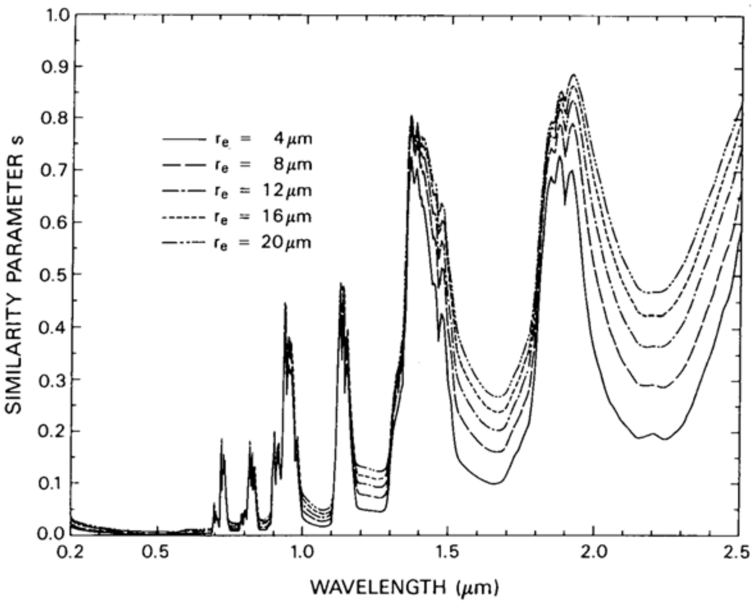
ω	s
1.0	0
0.999	0.081
0.99	0.25
0.9	0.65
0.8	0.79

$$\Delta A \rightarrow \Delta\sigma \propto -\Delta s \approx -c\,\Delta r_e$$

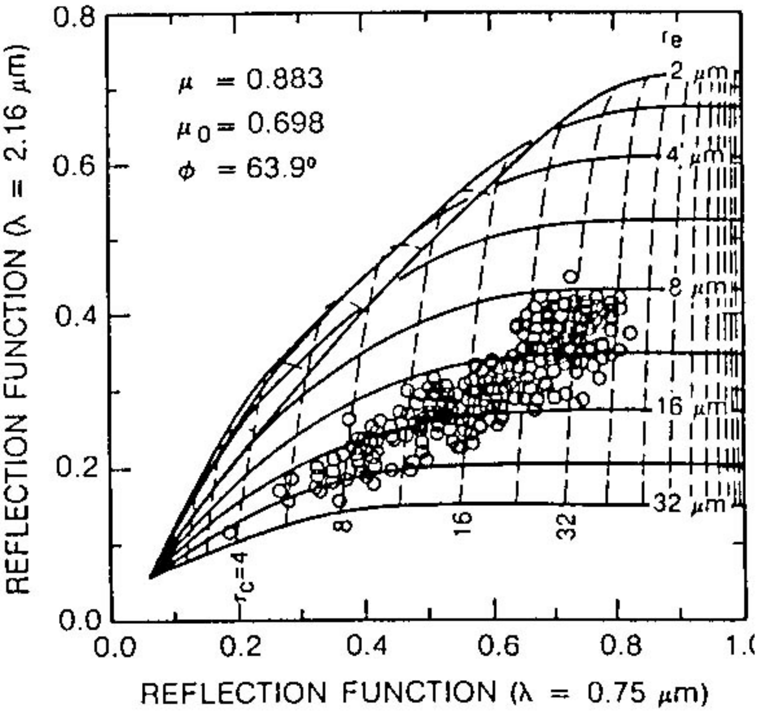
$$\sigma = \frac{1-s}{1+s} \qquad s = \sqrt{\frac{1-\omega}{1-\omega g}}$$

g=085

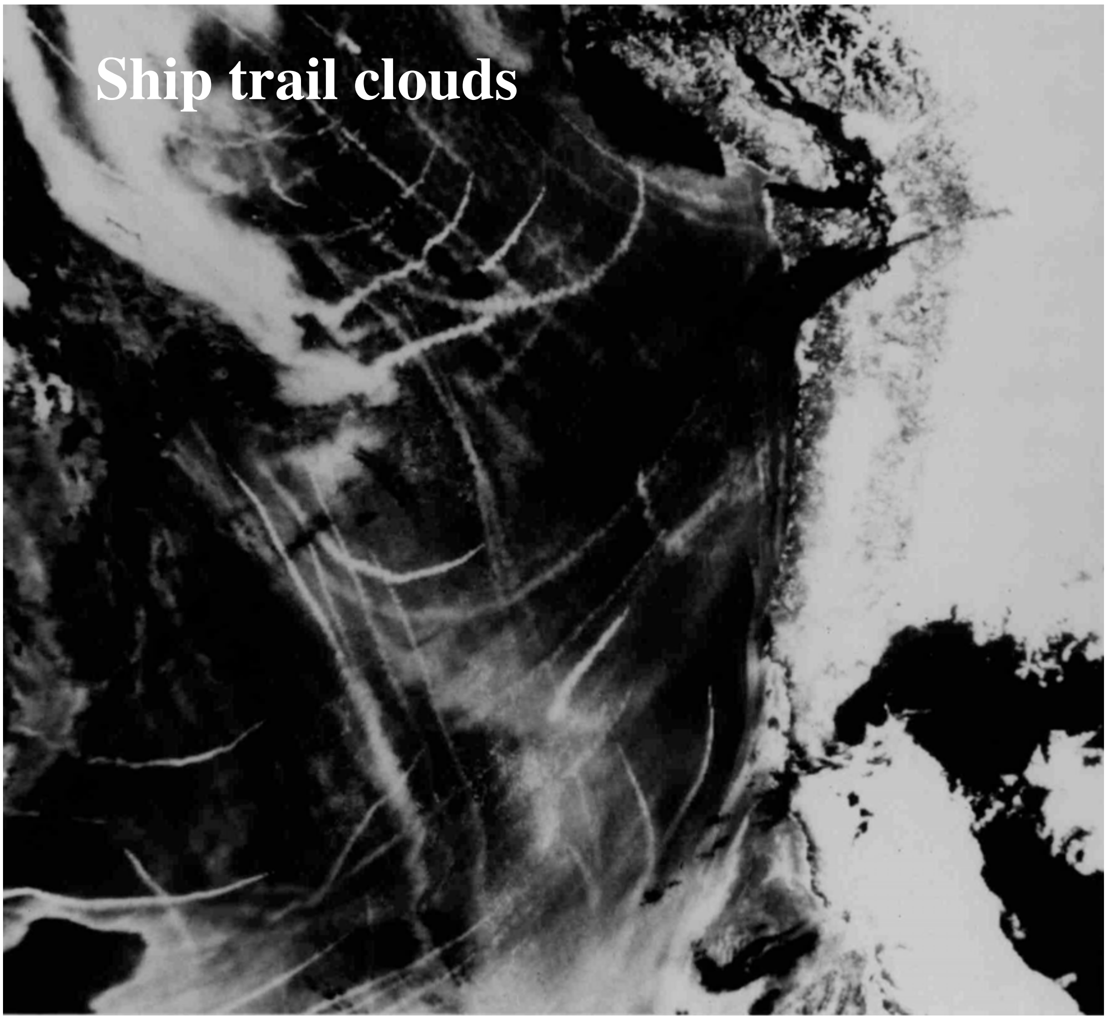
$$W = \frac{2\rho r_e \tau_c}{3}$$



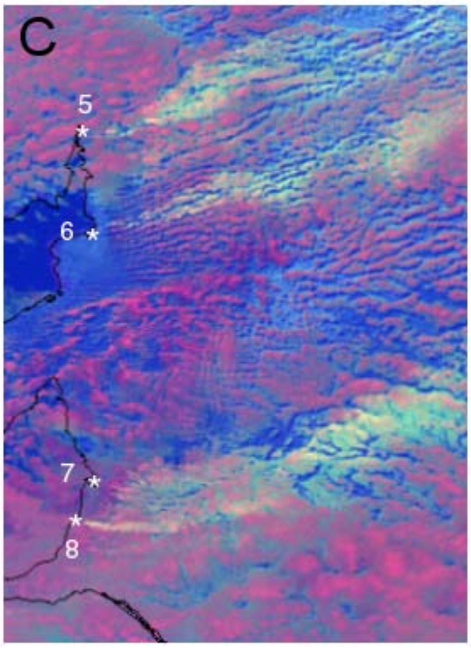
Solar reflection method for retrieving cloud optical properties



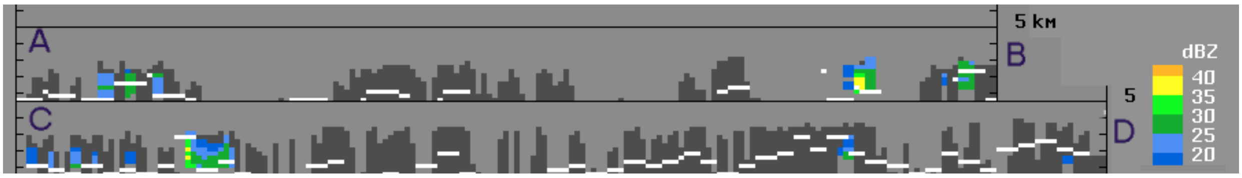
Ship trail clouds



Change in convective cloud properties



Rosenfeld (1999)



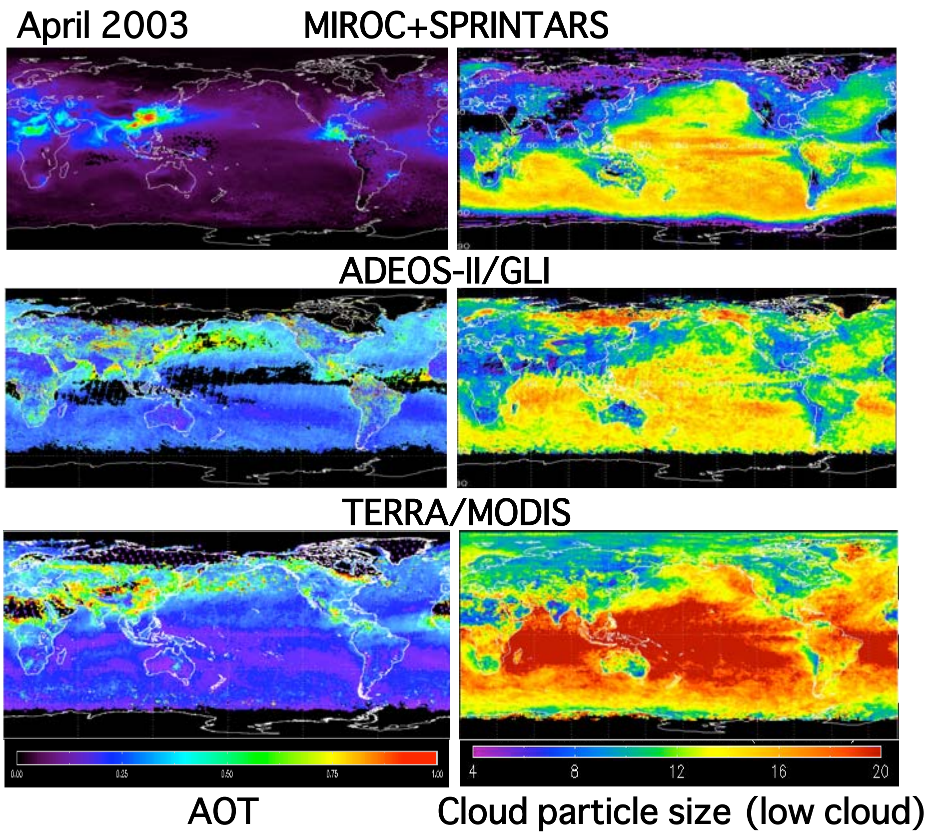
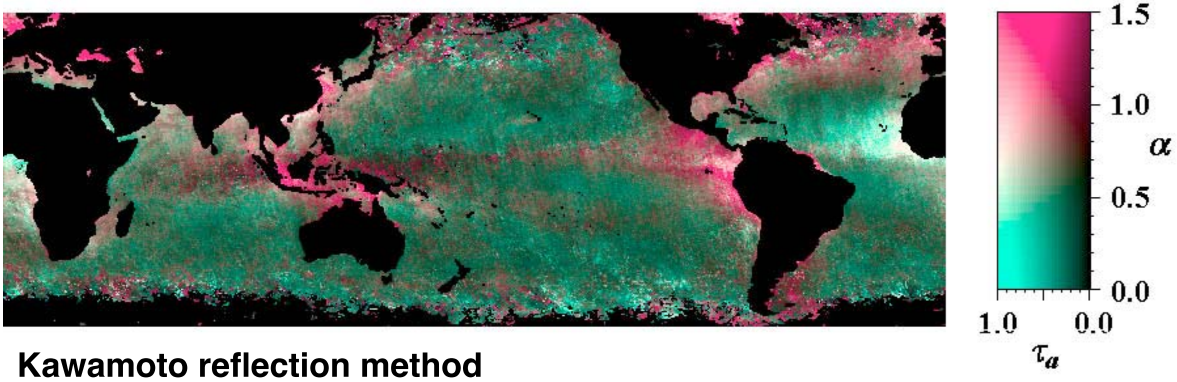


Fig. 3a. Comparison of Aerosol optical thickness (*AOT*) and effective cloud droplet radius (*CDR*) of low clouds from MIROC-GCM and two satellites (GLI and MODIS) (Nakajima and Schulz, 2009).

Large/small particulate distribution

Higurashi 2ch method

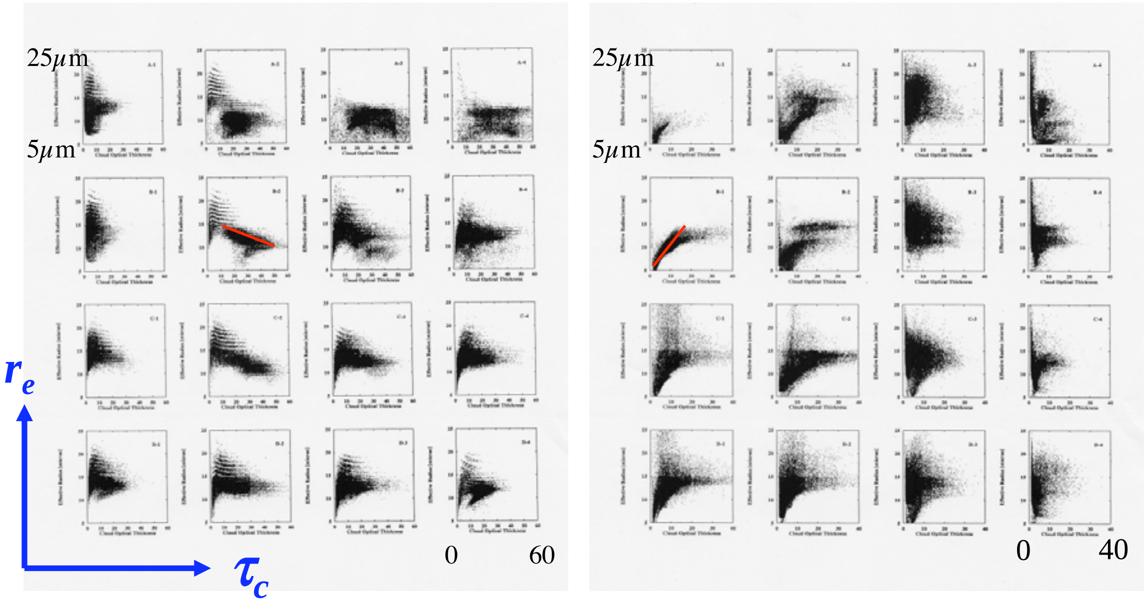


Kawamoto reflection method

Nakajima et al. (GRL 2001, p1171)

Correlation between r_e and τ_c

FIRE region (California) ASTEX region (North Atlantic)



Nakajima and Nakajima (JAS 1995)

→ Interpretation with model

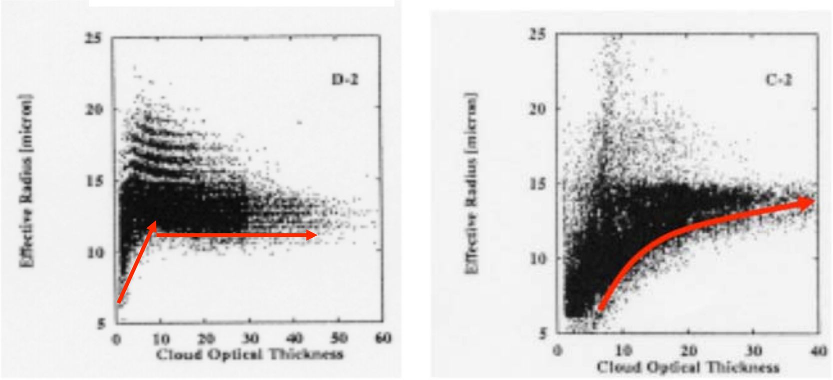
Effect of CCN addition on correlation pattern

Suzuki et al. (2006)

Satellite Obs.

Nakajima and Nakajima (JAS 1995)

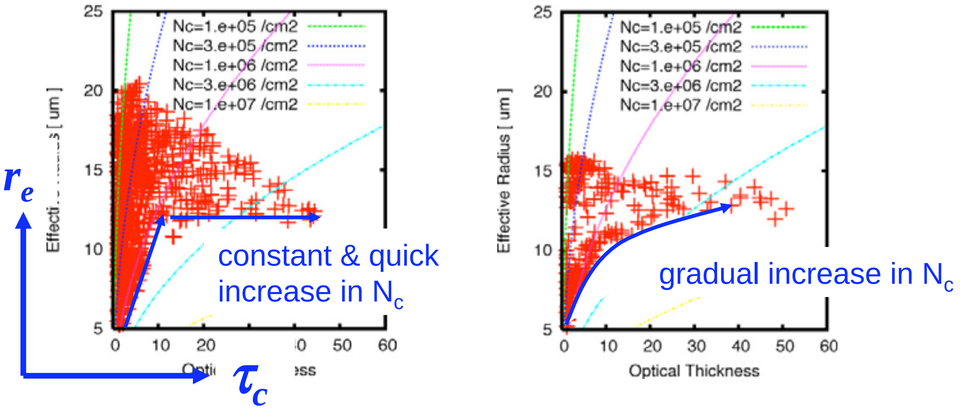
FIRE (Californian) ASTEX (North Atlantic)



Bin Model

$$\tau_c \approx 2\pi r_e^2 N_c$$

No CCN addition Continuous CCN addition



constant & quick increase in N_c

gradual increase in N_c

International Satellite Cloud Climatology Project (ISCCP)

- 4 geostational satellites and 2 polar orbiters
- Cloud reflectance and clear sky reflectance

$$n(\tau_c, T_c) \qquad\qquad A = nA_c + (1-n)A_s$$

Region	ISCCP	SOBS	METEOR	Nimbus-7
Global	62.6	61.5	60.9	52.9
NH	59.7	59.0	55.7	51.7
SH	65.4	64.0	66.0	54.1
Polar	52.3	68.6	50.4	58.0
Midlatitude	72.2	67.3	68.5	56.9
Tropics	58.4	55.4	58.2	48.5
Land	47.1	53.3	46.5	45.5
Ocean	70.2	65.5	67.9	56.5

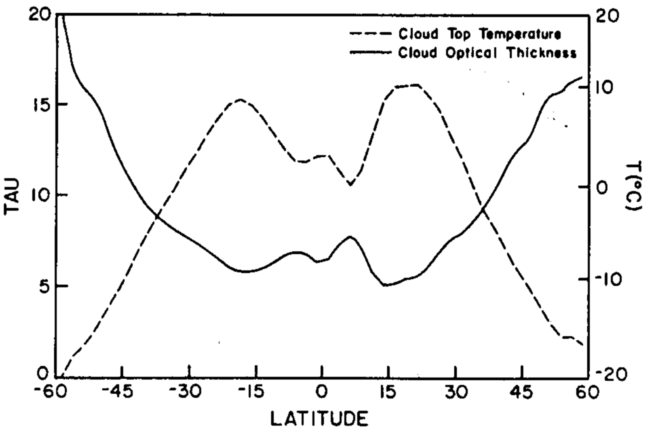


FIG. 2. Annual zonal-mean distributions of optical thickness and top temperature for all clouds in 1984.

Q

- Suppose the planetary albedo is *A*= 0.3. Calculate the global mean optical thickess of the cloud layer when *g*=0.85, the cloud amount *n*= 0.6 and clear sky reflectance *As*=0.1.
- Calculate the global mean cloud liquid water path (g/m2) if r_e=10 µm.

References

Charlson, R. J., S. E. Schwartz, J. M. Hales, R. D. Cess, J. A. Coakley, Jr., J. E. Hansen, and D. J. Hofmann, 1992: Climate forcing by anthropogenic aerosols. *Science*, **255**, 423-430.

Nakajima, T., M. D. King, J. D. Spinhirne, and L. F. Radke, 1991: Determination of the optical thickness and effective radius of clouds from reflected solar radiation measurements. Part II: Marine Stratocumulus Observations. *J. Atmos. Sci.*, **48**, 728-750.

Nakajima, T., A. Higurashi, K. Kawamoto, and J. E. Penner, 2001: A possible correlation between satellite-derived cloud and aerosol microphysical parameters. *Geophys. Res. Lett.*, **28**, 1171-1174.

Nakajima, T. Y., and T. Nakajima, 1995: Wide-area determination of cloud microphysical properties from NOAA AVHRR measurements for FIRE and ASTEX regions. *J. Atmos. Sci.*, **52**, 4043-4059.

Solution of the two stream equation 2

$$\frac{d^2\Psi(x)}{dx^2} = -(1-\omega g)(1-\omega)[-\Psi(x) + 2\pi B]$$

- **General solution for the homogeneous part**

$$\Psi(x) = C_+e^{kx} + C_-e^{-kx}$$

$$\Phi(x) = -\frac{1}{1-\omega g}\frac{d\Psi(x)}{dx} = -\frac{k}{1-\omega g}[C_+e^{kx} - C_-e^{-kx}] = -s[C_+e^{kx} - C_-e^{-kx}]$$

- **Diffusion exponent and similarity parameter**

$$k = \sqrt{(1-\omega g)(1-\omega)}, \quad s = \sqrt{\frac{1-\omega}{1-\omega g}}$$

- **Special solution for thermal emission**

$$\Psi(x) = \sum_n c_n x^n$$

Solution of the two stream equation 3

- **Special solution for thermal emission**

$$\sum_n n(n-1)c_n x^{n-2} = -(1-\omega g)(1-\omega)[-\sum_n c_n x^n + 2\pi B]$$

$$c_0 = 2\pi B, c_{n+1} = 0 \qquad \Psi = 2B, \quad \Phi = 0$$

$$\Psi = C_+ e^{kx} + C\_ e^{-kx} + 2\pi B, \quad \Phi = -s[C_+ e^{kx} - C\_ e^{-kx}]$$

$$F^+ = \frac{1}{2}[(1-s)C_+ e^{kx} + (1+s)C\_ e^{-kx}] + \pi B$$

$$F^- = \frac{1}{2}[(1+s)C_+ e^{kx} + (1-s)C\_ e^{-kx}] + \pi B$$

Standard problem

- **Boundary condition**

$$F^+(0) = F_0; \quad F^-(x_g) = 0$$

$$E = e^{kx_g}$$

$$(1-s)C_+ + (1+s)C\_ = 2(F_0 - \pi B)$$

$$(1+s)C_+ E + (1-s)C\_ E^{-1} = -2\pi B$$

$$C_+ = -2\frac{(1+s)\pi B + (F_0 - \pi B)(1-s)E^{-1}}{(1+s)^2 E - (1-s)^2 E^{-1}}$$

$$C_- = 2\frac{(1-s)\pi B + (F_0 - \pi B)(1+s)E^{-1}}{(1+s)^2 E - (1-s)^2 E^{-1}}$$

Solar radiation transfer 1

- **Thermal emission is small for λ<4 μm** $B=0$

$$C_- = -\frac{1+s}{1-s}E^2 C_+ \qquad F^+(x_1) = \frac{(1+s)^2-(1-s)^2}{(1+s)^2 E-(1-s)^2 E^{-1}}F_0$$

$$F^-(0) = \frac{(1-s^2)(E-E^{-1})}{(1+s)^2 E-(1-s)^2 E^{-1}}F_0$$

$$\sigma = \frac{1-s}{1+s}$$

$$t = \frac{1-\sigma^2}{1-\sigma^2 E^{-2}}E^{-1}$$

$$r = \frac{\sigma(1-E^{-2})}{1-\sigma^2 E^{-2}} = \sigma - \frac{\sigma(1-\sigma^2)}{1-\sigma^2 E^{-2}}E^{-2} = \sigma - \sigma t E^{-1}$$

Solar radiation transfer 3

- **Thick non-absorbing medium: Clouds in the visible region**

$$\omega \to 1, \quad s,k \to 0$$

$$\sigma \to 1-2s, \quad 1-\sigma^2 \to 4s, \quad E^{-1} \to 1-kx_g$$

$$t = \frac{1-\sigma^2}{1-\sigma^2 E^{-2}}E^{-1} \to \frac{4s}{(1+kx_g)-(1-4s)(1-kx_g)}$$

$$= \frac{4s}{2kx_g+4s} = \frac{1}{1+(1-g)x_g/2} = \frac{1}{1+bx_g} \qquad r = 1-t$$

  - **Optical thickness of the cloud layer is obtained from transmission or reflection measurements.**
  - **Cloud is bright to the human's eyes!**
  - **If ground albedo is included**

$$t = \frac{1}{1+(1-A_g)b\gamma\tau_c} \qquad r = 1-(1-A_g)t$$

Thermal emission 1

Infrared region without solar insolation $F_0 = 0$

Emissivity

$$F^+(0) = 0; \quad F^-(x_g) = 0 \qquad \omega < 1$$

$$C_+ = -2\frac{(1+s)-(1-s)E^{-1}}{(1+s)^2E-(1-s)^2E^{-1}}\pi B \qquad C_- = C_+E$$

$$\varepsilon = \frac{F^+(x_g)}{\pi B} = \frac{F^-(0)}{\pi B} = -[(1-s)E+(1+s)]\frac{(1+s)-(1-s)E^{-1}}{(1+s)^2E-(1-s)^2E^{-1}}+1$$

$$\varepsilon = 2s\frac{(1+s)-2E^{-1}+(1-s)E^{-2}}{(1+s)^2-(1-s)^2E^{-2}}$$

$$r + t + \varepsilon = 1$$

1991年（平成3年）3月30日（土曜日）

1日600万バレル 火勢衰えず

空覆う黒煙、有毒ガス

クウェート・ルポ

【アル・アハマディ（クウェート南部油田都市）二十九日＝当間敏雄】破壊された油井はまるで巨大なガスバーナーのようにゴウゴウとオレンジ色の火柱を噴き出し、煤煙（ばいえん）で暗く閉ざされた空を不気味に焦がした。百㍍ほどの距離に肉薄すると、火炎で顔がほてってくる。車のボディーやフロントガラスには空から降ってきた細かい油滴がタール状にこびりついた。記者は二十九日、クウェート最大の油田地帯「グレート・ブルガン」にある南部油田都市アル・アハマディを訪れ、破壊された油井を間近から取材、環境汚染の実態に迫った。

気温、10度以上も低下

サダム・フセイン・イラク大統領はクウェートの全油田を破壊するとの恐怖の予告を多国籍軍の地上戦開始と同時に実行に移したが、クウェート解放から一か月余りたった今も、イラク軍によって爆破、放火された約六百の油井の火勢は衰えず、大量の黒煙と有毒ガスを吐き出し続けている。

クウェート国営石油公社によると一日に炎上している原油の量は約六百万㌭。これはイラク軍侵攻以前のクウェートの原油生産量の二倍以上にも上る膨大なもの。クウェートの油床は内部圧力が高く、地表に噴き上げる力も強い。油井のバルブによって生産量を調整してきたものが、爆破によってバルブが吹き飛んだため、倍以上の原油が噴き出しているというのだ。

炎上する六百万㌭の原油は大量の煤煙となって膨大な黒雲を形成、日照を遮へいしており、南部の油田地帯は昼でも夜のように暗い。このため、気温も平年を十度以上も下回る肌寒さだ。クウェート市でも風向きによって黒雲にすっぽり覆われる日があり霧状になったタール分が浮遊、外を歩くとシャツがみるみる茶色くなる。

また、目に見える煤煙だけではなく、硫黄酸化物や窒素酸化物などの有毒ガスも大量に吐き出され続けているという。

病院関係者の話によると、油井の集中する南部地域では一部乳児に呼吸困難が見られるなど、住民への健康被害も懸念される状態となってきた。

クウェート公衆衛生局のラシド・アル・オワイシュ博士は二十八日、クウェート市内で記者会見し、「大気汚染による健康への差し迫った危険はない」と述べ、住民の不安解消への配慮を

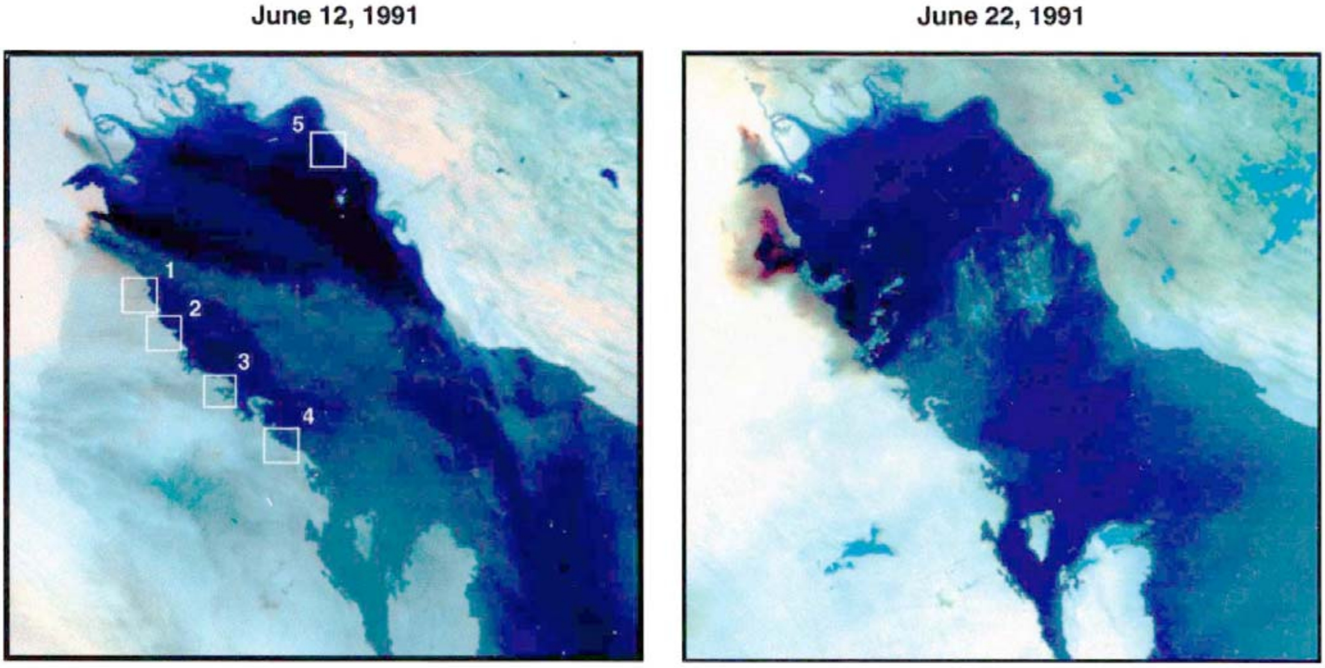
Persian Oil Fire Event in 1991

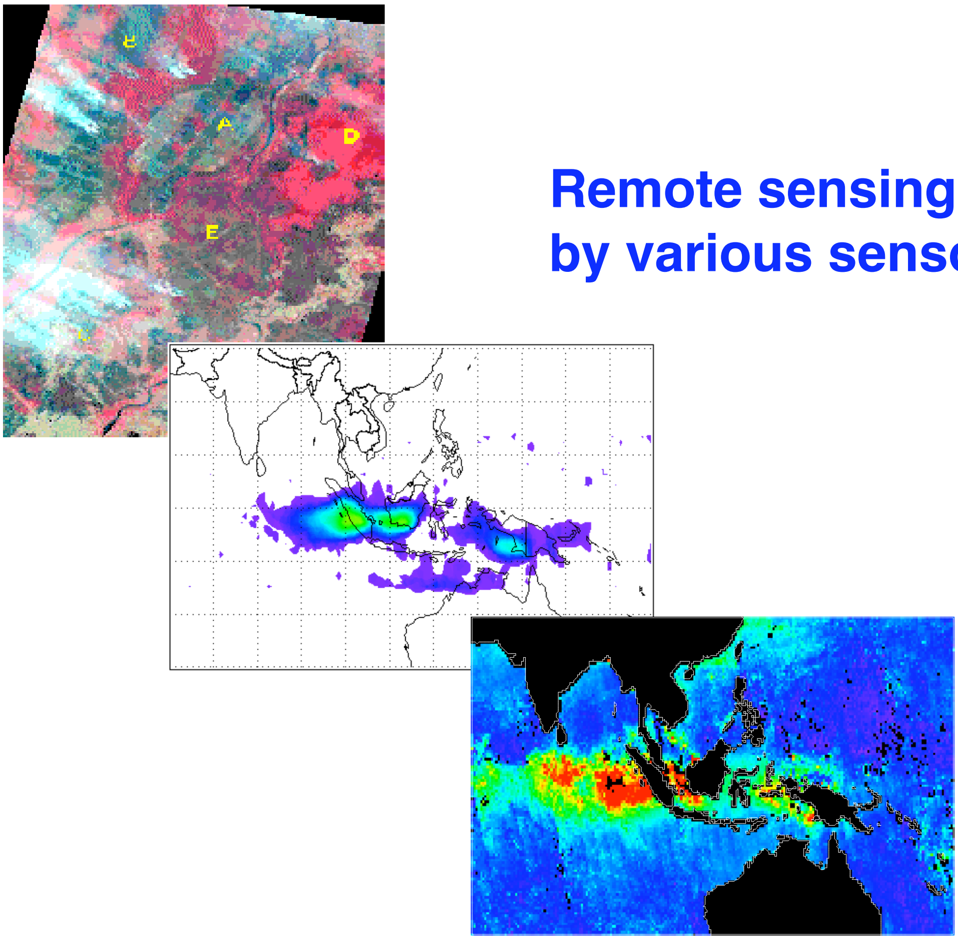
They have big one …

C131 R/Aircraft of U. Washington

We are scientists … in the Gulf war event in 1991

Oil fire smoke from sapce





Remote sensing of smoke by various sensors

APEX-E1 at Amami after 10 years